Online Appendix for Strategic Matching of Teachers and Schools with (and without) Accountability Pressure by Tom Ahn (thomas.ahn@uky.edu)

1 Theoretical Model

In this section, I define the teacher utility and school objective functions that must be satisfied for a match to be stable. In the model, I make the following assumptions:

Assumption 1 *A break-up of a match may be unilateral. A match must be mutually agreed upon.*

A match between school s and teacher a may be broken if either side wishes to search for a new match. A match between school s and teacher a must be mutually beneficial. In this model, this condition will be used to justify the stability of matches. For instance, assume it is observed in the data that school s is matched with teacher a and school s' is matched with teacher a'. This match would be unstable (and thus not observed) if one school (say s') *and* one teacher (say a) find that they would rather be paired compared to the original match. If one teacher and one school are in agreement, the original coalition is broken, and s' matches with a.[1]

Assumption 2 *A teacher cannot offer inducements to another teacher to swap matches, and a school cannot offer inducements to another school to swap matches. However, a school may offer (non-monetary) inducements to a teacher, and a teacher may offer (non-monetary) inducements to a school.*[2]

In this application, I assume that the inducements are reductions in (as inducements from the school side) or volunteering for increased (as inducements from the teacher side) administrative work or unattractive assignments that is unobservable to the econometrician. Unattractive assignments are any disadvantageous allocations of scarce resources, such as increased administrative workload, poorly maintained (or temporary) classrooms, outdated equipment, lack of help from teaching assistants, and lack of mentoring. Research has shown that principals may be strategically assigning teachers into classrooms with differing attributes

[1] No assumption is made about s and a' matching in this case.

[2] See Kelso and Crawford (1982) for a similar description of the formation of a coalition.

as rewards or incentives to stay at the school or to increase education production. (See Clotfelter, Ladd, and Vigdor (2005) or Kalogrides, Loeb, and Beteille (2013) for examples.)

Consider a finite set of teachers A and a finite set of schools S. Define the utility gain of teacher a moving from his or her original school s_o to the new school s as $u_a(s, s_o) + \epsilon_{a,s}$, where vector s contains pecuniary and non-pecuniary characteristics of the school including salary and working conditions (and commuting distance), and $\epsilon_{a,s}$ is a match-specific error. Similarly, define the marginal utility gain of a school (or principal) s having teacher a on staff compared to the outside option 0 (in this case a newly minted teacher) as $g_s(a) + \epsilon_{s,a}$.[3] The joint production (or utility) function of the teacher-school match is:

$$f(s, s_o, a) + \eta_{a,s} = u_a(s, s_o) + g_s(a) + \eta_{a,s}$$

where $\eta_{a,s} = \epsilon_{a,s} + \epsilon_{s,a}$.[4]

One limitation in this study to note is that portions of the joint utility function are not exclusively attributable to the teacher or the principal. For instance, a teacher with high experience may be matched more frequently with schools that have higher test scores because 1) experienced/older teachers prefer to teach better performing students, 2) schools that have high achievement may (correctly or incorrectly) believe that experienced teachers will maintain or increase performance, or 3) both. Because of this, the observed match (and the estimated utility production) is always assumed to be joint, inseparable, and specific to the match.

Following Fox (2009), if a feasible teacher-school assignment is defined as $A - S_1$ and the same assignment with *any pair* of teachers (a, a') and schools (s, s') matches flipped defined as $A - S_2$, with some data of teachers and schools X, I assume rank-ordering as follows:[5]

[3] I remain agnostic about the principal's utility function, by *not* assuming that she seeks to maximize educational achievement. Some alternative utility maximizing solutions may be to minimize the level of classroom disruption at the school through recruitment of experienced teachers or minimize potential effort exertion by recruiting less ambitious teachers. It may also make sense to think of the principal and the remaining teachers at the school as one coalition with a common utility function.

[4] The joint error term η can be interpreted as a match-specific error term, which could be, for instance, personal/professional compatibility between the principal and the teacher during the recruiting process.

[5] With the above assumption, general results from Fox (2009) can be used to show that the probability of any hypothetical market-wide teacher-school assignment is equal to the integration of an indicator function of the particular assignment maximizing the joint output of all matches over the error distribution, given some intial parameter guess. While the identity defined is not exactly what is required to integrate an error term

Assumption 3 *Rank-order property:*

$$f(s, s_o, a) + f(s', s'_o, a') \geq f(s, s_o, a') + f(s', s'_o, a)$$

if and only if:

$$Pr(A - S_1|X) \geq Pr(A - S_2|X)$$

Finally, I define pairwise stability as follows:

Definition *Pairwise Stability: An assignment of a teacher to a school is pairwise stable if for every pair of teachers* $(a, a') \in A$ *and every pair of schools* $(s, s') \in S$*, the following condition holds:*

$$f(s, s_o, a) + f(s', s'_o, a') \geq f(s, s_o, a') + f(s', s'_o, a)$$

While noting that the joint production $f(\cdot)$ is not easily separable in the data, assume that it is possible theoretically to attribute all portions of $f(\cdot)$ to either the teacher (u) or the principal (g). Assume that teacher a is 'more desirable' to the principal at school s' compared to the current match, teacher a'. The principal at school s' can offer inducement t to teacher a to swap schools by offering less administrative work or more attractive assignments (assuming of course, that he or she can 'afford' to do so). Note that administrative work *must* be completed by someone and equipment must be purchased at cost t.[6] If a match is pairwise stable, I assume that there does not exist an exchange of matches that improves any of the principals' or teachers' outcomes. That is:

into MSE (as the error is defined at the match-level and not at the assignement-level) and rely on Manski (1975)'s single-agent multi-nomial results, Monte-Carlo simulations show that error defining the probability in the above fashion is a close approximation of the rank ordering assumption.

[6]It is worth emphasizing that teacher a' is not working at school s' in the current year. Both teachers and both schools are in the transfer market, and the current pairing of a to s and a' to s' is for ease of exposition.

$$
\begin{aligned}
u_a(s, s_o) &\geq u_a(s', s_o) + t_{as'} \\
&\text{or} \\
g_{s'}(a') &\geq g_{s'}(a) - t_{as'} \\
&\text{and} \\
u_{a'}(s', s'_o) &\geq u_{a'}(s, s'_o) - t_{a's} \\
&\text{or} \\
g_s(a) &\geq g_s(a') + t_{a's}
\end{aligned}
$$

The term $t_{as'}$ ($t_{a's}$) is any value in the range of the minimum transfer value that would make teacher a (a') indifferent between s and s' and the maximum amount that would make the principal at school s' (s) indifferent between teachers a and a'. Therefore, these four conditions are equivalent to saying that at least one teacher or one principal from each coalition opposes swapping matches.

To expand, assume that t^* is the transfer value that makes teacher a indifferent between the two schools, s and s'. Then I assume:

$$
\begin{aligned}
u_a(s, s_o) &= u_a(s', s_o) + t^* \\
g_{s'}(a') &\geq g_{s'}(a) - t^*
\end{aligned}
$$

which means:

$$u_a(s, s_o) + g_{s'}(a') \geq u_a(s', s_o) + g_{s'}(a)$$

This is equivalent to saying that the benefits that must be offered to the other teacher is so large that the principal opts not to seek to break the coalition. The flip side is the following condition:

$$u_{a'}(s', s'_o) + g_s(a) \geq u_{a'}(s, s'_o) +_s (a')$$

This condition is equivalent to saying that the amount of administrative duties that must be taken on by the teacher who is considering the swap is so great that she opts not to break the original match. The summation of the two conditions (and re-introducing the joint production framework), we have the pairwise stability condition.

As abstract as the theory model is, the actual transfer process in North Carolina resembles the outlines of the model. Indeed, it seems that principals and teachers actively search for each other. The hiring of new teachers and transfers in North Carolina is handled both at the school level and the district level. While the vacancies are advertised at the school level, employment lists are maintained centrally by the district.[7] From discussions with principals and education policy practitioners in North Carolina, it became clear that principals who are motivated to find the 'best' candidates aggressively search through district lists to look for good applicants. In addition, ambitious applicants are known to contact principals directly or through other word-of-mouth means, such as seeking exposure while working as a student teacher (for new teachers) or 'lobbying' through colleagues at their desired schools (for transfers). Some districts have attempted to prevent such aggressive principals from recruiting. For instance, Wake County limits the number of teachers principals can take with them, as they move to new assignments. The policy was implemented in response to parent complaints that exiting principals were poaching the most effective teachers. Since some principals actively recruit teachers, the model set-up will clarify which principals search, and what types of teachers they search for.

It should be noted that the initial decision to search by teachers is taken as given, and in particular I do not model how the expected matching outcome generated by the model in turn influences the initial search decision. While this is a limitation of the model, in this paper, I am interested in what the stable distribution of teachers across schools with different accountability pressures looks like, conditional on teachers searching.[8] In addition, the modest bonus amount (of \$750 or \$1,500) seems unlikely to spur a major decision to move to a new school.

[7]There seems to be more than one outlet to advertise teaching positions in North Carolina. One prominent example is http://schooljobs.dpi.state.nc.us/Jobs/Search.

[8]See Ahn (2015) for a model that accounts for the initial search decision by a teacher (and how it affects all other teachers' search decisions.

2 Tables

Table 1: Accountability History

# of Times Bonus Received	% of Schools
5	34.50%
4	27.90%
3	23.30%
2	9.60%
1	3.30%
0	1.30%

Five year accountability history of schools (2002 - 2007).

Table 2: Transition Matrix between High and Low-stakes Grades

		Year t+1	
		low-stakes	high-stakes
Year t	low-stakes	0.9385	0.0615
	high-stakes	0.1296	0.8703

Table 3: Characteristics of Transitioning Teachers

	high - high	high - low	low - low	low - high
female	0.9363	0.9373	0.9244	0.9306
minority	0.1544	0.1373	0.1403	0.1679
no fixed effect	0.0253	0.0778	0.0235	0.0835

Table 4: Estimates of Education Production

Variable	Reading Score	Math Score
last yr. score	0.7066 (0.0006)	0.7399 (0.0005)
female	0.0377 (0.0010)	0.0032 (0.0009)
minority	-0.1637 (0.0013)	-0.1431 (0.0012)
parent low ed.	-0.1748 (0.0012)	-0.1564 (0.0011)
class size	-0.0032 (0.0002)	-0.0047 (0.0002)
tch. female	0.0066 (0.0275)	0.0204 (0.0253)
tch. minority	-0.0354 (0.0184)	-0.0012 (0.0169)
experience	0.0027 (0.0012)	0.0030 (0.0011)
experience^2	-0.0002 (0.0001)	-0.0003 (0.0001)
certified	-0.0105 (0.0046)	-0.0094 (0.0042)
peer female %	0.0323 (0.0068)	0.0448 (0.0062)
peer minority %	-0.0589 (0.0073)	-0.0622 (0.0067)
school size/1000	-0.0132 (0.0079)	-0.0199 (0.0001)
school minority %	0.0130 (0.0126)	0.0361 (0.0115)
school rural	-0.0064 (0.0030)	-0.0029 (0.0028)
Observations	1,498,471	1,507,394
FE	2.14e-11 (0.1693)	2.59e-11 (0.2434)

NCERDC data from 1998/99 to 2002/03. FE is fixed effect. Low parental education is HS degree or below.

Table 5: Estimates of Matching Parameters at half bandwidth†

Variable	Estimate (treatment)	95% CI	Estimate (control)	95% CI
		Teacher-side		
Δ distance %	-1	super-consistent	-1	super-consistent
Δ urbanicity	0.2917	(-4.2999 , 4.8833)	1.2304	(-1.3707 , 3.8315)
Δ classroom minority %	-16.3041	(-21.0058 , -11.6024)	-26.7427	(-29.5563 , -23.9291)
Δ school minority %	-15.6027	(-17.057 , -14.1484)	-23.8347	(-26.7396 , -20.9298)
Δ classroom minority % X teacher minority	19.8805	(15.0759 , 24.6851)	-4.0252	(-4.8562 , -3.1942)
Δ school minority % X teacher minority	19.766	(18.3117 , 21.2203)	-3.1669	(-4.1593 , -2.1745)
Δ level achievement	-4.7151	(-6.8779 , -2.5523)	-5.6782	(-7.6707 , -3.6857)
Level achievement X experience	27.7535	(25.2621 , 30.2449)	23.5859	(22.0487 , 25.1231)
Level achievement X math FE	26.3872	(24.007 , 28.7674)	22.0942	(20.2173 , 23.9711)
Level achievement X certification	16.8675	(14.5139 , 19.2211)	8.4378	(6.735 , 10.1406)
Level achievement X teacher minority	-16.9034	(-19.5703 , -14.2365)	-2.1815	(-3.8463 , -0.5167)
ABC pressure X experience	16.2968	(13.2945 , 19.2991)	6.4012	(5.0915 , 7.7109)
ABC pressure X math FE	15.4681	(12.2857 , 18.6505)	-0.7207	(-1.9654 , 0.524)
ABC pressure X certification	6.0194	(3.5066 , 8.5322)	-0.6	(-1.7911 , 0.5911)
ABC pressure X teacher minority	8.4168	(5.1048 , 11.7288)	2.8049	(1.4546 , 4.1552)
Level X ABC X experience	20.1951	(17.8478 , 22.5424)	25.4919	(24.2354 , 26.7484)
Level X ABC X math FE	18.6013	(16.302 , 20.9006)	24.1063	(22.8915 , 25.3211)
Level X ABC X certification	8.9186	(6.6508 , 11.1864)	10.0921	(8.895 , 11.2892)
Level X ABC X teacher minority	-16.3491	(-18.9396 , -13.7586)	-15.3683	(-16.6686 , -14.068)
h (bandwidth)		0.5635		

†Experience and fixed effects are converted to percentile values. Level achievement is average math proficiency rate at the school-level.

Table 6: Estimates of Matching Parameters at double bandwidth[†]

Variable	Estimate (treatment)	95% CI	Estimate (control)	95% CI
		Teacher-side		
Δ distance %	-1	super-consistent	-1	super-consistent
Δ urbanicity	-0.712	(-7.1903 , 5.7663)	-1.301	(-3.5892 , 0.9872)
Δ classroom minority %	-16.6591	(-23.5547 , -9.7635)	-25.5518	(-28.4915 , -22.6121)
Δ school minority %	-12.4842	(-13.2392 , -11.7292)	-26.1406	(-29.2205 , -23.0607)
Δ classroom minority % X teacher minority	18.6496	(11.5749 , 25.7243)	2.6434	(1.8662 , 3.4206)
Δ school minority % X teacher minority	20.3022	(19.5472 , 21.0572)	-2.2801	(-3.1063 , -1.4539)
Δ level achievement	-6.6851	(-10.4087 , -2.9615)	-4.8311	(-6.8259 , -2.8363)
Level achievement X experience	24.1181	(20.1838 , 28.0524)	24.8476	(23.4928 , 26.2024)
Level achievement X math FE	27.4209	(23.5777 , 31.2641)	26.0206	(24.3215 , 27.7197)
Level achievement X certification	15.4257	(11.7684 , 19.083)	12.069	(10.4195 , 13.7185)
Level achievement X teacher minority	-17.7221	(-21.5768 , -13.8674)	-4.9281	(-6.4182 , -3.438)
ABC pressure X experience	15.345	(10.8974 , 19.7926)	8.3549	(7.0941 , 9.6157)
ABC pressure X math FE	15.3431	(10.2163 , 20.4699)	-5.1958	(-6.5844 , -3.8072)
ABC pressure X certification	8.5537	(4.8052 , 12.3022)	2.8014	(1.5241 , 4.0787)
ABC pressure X teacher minority	9.2769	(4.2299 , 14.3239)	4.384	(2.9781 , 5.7899)
Level X ABC X experience	21.3611	(17.7288 , 24.9934)	26.2716	(25.1453 , 27.3979)
Level X ABC X math FE	18.2931	(14.5381 , 22.0481)	25.8632	(24.5242 , 27.2022)
Level X ABC X certification	10.0714	(6.5543 , 13.5885)	14.9942	(13.7642 , 16.2242)
Level X ABC X teacher minority	-19.6802	(-23.4354 , -15.925)	-16.0872	(-17.3289 , -14.8455)
h (bandwidth)		2.254		

[†]Experience and fixed effects are converted to percentile values. Level achievement is average math proficiency rate at the school-level.

Table 7: Estimates of Matching Parameters with Intr-District Transfers Only[†]

Variable	Point Estimate	95% CI
Teacher-side		
Δ distance %	-1	super-consistent
Δ urbanicity	-3.9982	(-9.4829 , 1.4865)
Δ classroom minority %	-17.2341	(-22.6701 , -11.7981)
Δ school minority %	-12.7087	(-16.7341 , -8.6833)
Δ classroom minority % X teacher minority	11.4425	(5.9839 , 16.9011)
Δ school minority % X teacher minority	29.7103	(25.6889 , 33.7317)
Δ level achievement	-4.7196	(-9.0956 , -0.3436)
Level achievement X experience	20.5722	(16.7652 , 24.3792)
Level achievement X math FE	29.7515	(26.1217 , 33.3813)
Level achievement X certification	23.2207	(18.3068 , 28.1346)
Level achievement X teacher minority	-21.3918	(-26.2383 , -16.5453)
ABC pressure X experience	5.7138	(2.2931 , 9.1345)
ABC pressure X math FE	4.9028	(-0.1917 , 9.8250)
ABC pressure X certification	6.9701	(3.3889 , 10.5513)
ABC pressure X teacher minority	7.1572	(3.7296 , 10.5848)
Level X ABC X experience	15.8396	(12.617 , 19.0622)
Level X ABC X math FE	6.7542	(3.7155 , 9.7929)
Level X ABC X certification	3.0626	(-0.4363 , 6.5615)
Level X ABC X teacher minority	-30.0116	(-33.3382 , -26.685)
h (bandwidth)	1.366	

[†]Experience and fixed effects are converted to percentile values. Level achievement is average math proficiency rate at the school-level.

Table 8: Estimates of Matching Parameters with 2002/03 Sample Excluded[†]

Variable	Point Estimate	95% CI
Teacher-side		
Δ distance %	-1	super-consistent
Δ urbanicity	-3.868	(-6.8295 , -0.9065)
Δ classroom minority %	-7.0629	(-9.3692 , -4.7566)
Δ school minority %	-7.2226	(-9.0392 , -5.406)
Δ classroom minority % X teacher minority	21.4377	(19.1319 , 23.7435)
Δ school minority % X teacher minority	19.8747	(18.0579 , 21.6915)
Δ level achievement	0.1164	(-2.0764 , 2.3092)
Level achievement X experience	18.204	(16.8264 , 19.5816)
Level achievement X math FE	28.1991	(26.851 , 29.5472)
Level achievement X certification	21.6601	(20.5634 , 22.7568)
Level achievement X teacher minority	-12.8378	(-14.295 , -11.3806)
ABC pressure X experience	4.975	(2.9987 , 6.9513)
ABC pressure X math FE	4.9121	(2.8852 , 6.939)
ABC pressure X certification	12.7146	(11.5051 , 13.9241)
ABC pressure X teacher minority	5.2897	(3.1525 , 7.4269)
Level X ABC X experience	14.1353	(12.7883 , 15.4823)
Level X ABC X math FE	11.9452	(10.6245 , 13.2659)
Level X ABC X certification	7.6802	(6.6651 , 8.6953)
Level X ABC X teacher minority	-20.9315	(-22.3591 , -19.5039)
h (bandwidth)	1.701	

[†]Experience and fixed effects are converted to percentile values. Level achievement is average math proficiency rate at the school-level.

References

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