S1 The Response Strategies Integrated in MOEA-OSD/SRS and SADI

This section introduces six response strategies, i.e. RDI, MDI, LPS, PPS, FPS and SADI in detail.

S1.1 Random Diversity Introduction (RDI)

Random diversity introduction (RDI) is described as a certain percentage of individuals in population are replaced by the individuals generated randomly when an environmental change is detected. Since this approach introduces new solutions, it can enlarge the search space and may be good at improving performance in environments with severe changes.

S1.2 Mutational Diversity Introduction (MDI)

Mutational random diversity introduction (MDI) is described as a certain percentage individuals of the population are mutated when an environmental change is detected. Since the new individuals are related to the individuals of the current population, this approach may be beneficial for small environmental changes.

S1.3 Linear Prediction Strategy (LPS)

Linear prediction strategy (LPS) predicts the new position of individual according to the previous position change when an environmental change is detected. Assuming that the historical Pareto optimal solutions are $Q_1, ..., Q_1$ in the previous time windows, and $x_1, x_2, ... x_t$, $x_t \in Q_i$, $i = 1, ..., t$ are a series of points in the decision space, a model for predicting the new
position of an individual in $Q_{t+1}$ is shown as follows:

$$x_{t+1} = F(x_t, x_{t-1}) = x_t + (x_t - x_{t-1})$$  \(1\)

where $K$ is the number of the previous time windows. For a point $x_t \in Q_t$, its position in $Q_{t-1}$ can be defined as the closest point from $x_t$ in $Q_{t-1}$, i.e.,

$$x_{t-1} = \arg\min_{y \in Q_{t-1}} \| y - x_t \|_2$$  \(2\)

A linear model is used to predict the position of an individual for the next time window according to Eq. (3):

$$x_{t+1} = F(x_t, x_{t-1}) = x_t + (x_t - x_{t-1})$$  \(3\)

Adding a predicted Gaussian noise to the predicted positions may increase the probability of the initial population to close to the PS. The Gaussian noise is described as follows:

$$\varepsilon \sim N(0, \delta I)$$  \(4\)

where $\delta$ is the standard deviation, which is estimated according to the information from the previous occurred changes and it can be computed as follows:

$$\delta^2 = \frac{1}{4n} \| x_t - x_{t-1} \|_2^2$$  \(5\)

According to the above description, the following linear model is used to predict the new position of the individual in the new time window:

$$x_{t+1} = F(x_t, x_{t-1}) = x_t + (x_t - x_{t-1}) + \varepsilon$$  \(6\)

### S1.4 Population Prediction Strategy (PPS)

In PPS, a Pareto set is divided into two parts: a center point and a manifold. Using a set of center points in previous times estimates the new center, and the previous manifolds are used to predict the new manifold in next time. Thus, in the new environment, the whole initialization population is formed by combining the predicted center and manifold.

At the moment $t$, the solutions set $POP^t$ can be described as a combination of a center $C_t$ and a manifold $M_t$ as follows:

$$POP^t = C_t + M_t$$  \(7\)

After the center $C_{t+1}$ and the manifold $M_{t+1}$ of $(t+1)$–$th$ environment are predicted, they are combined to generate the initial population of the $(t+1)$–$th$ moment.

#### S1.4.1 Predict the Center $C_{t+1}$

Firstly, An autoregression (AR) model is used to predict the center of $PS$ of the $(t+1)$–$th$ environment, the model is denoted as AR (p) model and the order of the model is set as $p=3$, AR (3) is described as:

$$C_{t+1} = \theta_1 C_t + \theta_2 C_{t-1} + \theta_3 C_{t-2} + \varepsilon_t$$  \(8\)

where $C_t = (C_{t,1}, C_{t,2}, ..., C_{t,D})^T$, $\theta_j = (\theta_{j,1}, \theta_{j,2}, ..., \theta_{j,D})^T$, $j = 1, 2, 3$ are the parameters in AR(3), $\varepsilon_t = (\varepsilon_{t,1}, \varepsilon_{t,2}, ..., \varepsilon_{t,D})^T$ is a white noise, $\varepsilon_{t,j} \sim N(0, \sigma_{t,j}^2)$, and $\sigma_{t,j}^2 = (\sigma_{1,j}^2, \sigma_{2,j}^2, ..., \sigma_{D,j}^2)^T$. Considering the $i$-$th$ dimension, and setting the length of time series is $M = 23$, the parameters can be calculated as:
\begin{align*}
C_i^t &= \theta_1^i C_i^{t-1} + \theta_2^i C_i^{t-2} + \theta_3^i C_i^{t-3} \\
C_i^{t-1} &= \theta_1^i C_i^{t-2} + \theta_2^i C_i^{t-3} + \theta_3^i C_i^{t-4} \\
&\vdots \\
C_i^{t-20} &= \theta_1^i C_i^{t-21} + \theta_2^i C_i^{t-22} + \theta_3^i C_i^{t-23}
\end{align*}

Let \( \Psi_i = (C_i^t, C_i^{t-1}, \ldots, C_i^{t-20})^T \), and
\[
\Phi_i = \begin{bmatrix}
C_i^{t-1} & C_i^{t-2} & C_i^{t-3} \\
C_i^{t-2} & C_i^{t-3} & C_i^{t-4} \\
\vdots & \vdots & \vdots \\
C_i^{t-21} & C_i^{t-22} & C_i^{t-23}
\end{bmatrix}
\]

Eq. (8) can be abbreviated as:
\[
\Psi_i = \Phi_i (\theta_1^i, \theta_2^i, \theta_3^i)^T
\]

The least squares regression method is used to calculate \( \theta \), as described in Eq. (13).
\[
(\theta_1^i, \theta_2^i, \theta_3^i)^T = (\Phi_i^T \Phi_i)^{-1} \Phi_i^T \Psi_i
\]

where, the parameter \( \sigma 1_i^k \) is the average squared error, shown as follows:
\[
\sigma 1_i^k = \frac{1}{M-p} \sum_{k=t-M+p}^{t} [C_i^k - \theta_1^i C_i^{k-1} - \theta_2^i C_i^{k-2} - \theta_3^i C_i^{k-3}]
\]

**S1.4.2 Predict the Manifold \( M^{t-1} \)**

PPS uses the manifold of the last two moments, i.e. \( M^t \) and \( M^{t-1} \), to predict the \( M^{t+1} \). Considering the \( i \)-th dimension, the manifold \( M_{i+1} \) at the moment \( t+1 \) is predicted by adding a Gaussian error \( \varepsilon_i^t \) on \( M_i^t \), as shown in Eq.14:
\[
M_{i+1}^t = M_i^t + \varepsilon_i^t
\]

where \( i = 1, 2, \ldots, n \) and \( \varepsilon_i^t \sim N(0, \sigma 2_i^t) \), \( \sigma 2_i^t \) is given as follows:
\[
\sigma 2_i^t = \frac{1}{n} D(M^t, M^{t-1})^2
\]

where \( D(M^t, M^{t-1}) \) is the distance between manifolds \( M^t \) and \( M^{t-1} \), which is calculated as follows:
\[
D(M^t, M^{t-1}) = \frac{1}{|M^t|} \sum_{x \in M^t} \min_{y \in M^{t-1}} \|x - y\|
\]

Finally, the combination of \( C^{t+1} \) and \( M^{t+1} \) form the initial population of the \( (t + 1) - th \) environment, i.e. \( POP^{t+1} \).
\[
POP^{t+1} = C^{t+1} + M^{t+1}
\]
S1.5 Feed-Forward Prediction Strategy (FPS)

After detecting an environmental change, the position of the optimal solution of the new environment is estimated by using the autoregressive model in FPS. The initial population consists of three groups of individuals: the first group includes \(3(m+1)\) individuals which are predicted by using FPS, (where \(m\) is the number of the objective function), the second group is 70% of the remaining individuals, and they were randomly selected from the previous population, and 30% of the remaining individuals are randomly generated. For instance, the population size is 100, the number of the objective function is 2, so the number of individuals to be predicted is 9, 64 individuals are selected randomly from the previous population, and 27 individuals are generated randomly. FPS also uses the AR (p) model to predict the \(3(m+1)\) individuals of the next environment.

S1.6 Self-adaptive Diversity Introduction (SADI)

In SADI, the extent of environmental change is estimated according to the following equation at first:

\[
\delta(t) = \sum_{i=1}^{n_s} \left( \left\| F(x^i, k) - F(x^i, k-1) \right\| \right) \quad (18)
\]

where \(n_s\) represents the \(n_s\) individuals chosen from the current population randomly. \(F(x^i, k)\) and \(F(x^i, k + 1)\) are the objective function values of the individual \(x^i\) between two adjacent generations \(k\) and \(k + 1\).

Secondly, according to the extent of environmental change we can calculate diversity introduction ratio approximately:

\[
\zeta(t) = \eta(\delta(t)) = \min(\lambda \times \frac{\delta(t) - \delta_{\text{min}}}{\delta_{\text{max}} - \delta(t)}, 1.0) \quad (19)
\]

where \(\zeta(t)\) is the diversity introduction ratio, \(\delta(t)\) is the extent of environmental change. \(\delta_{\text{min}}\) and \(\delta_{\text{max}}\) are the maximum extent of environmental change and the minimum extent of environmental change respectively. Here the scale factor is set as \(\lambda = m - 1\).

Then, an adaptive relocated operator is designed to generate the initial population after changes occur. The probability of random initialization is \(\zeta(t)\), and the probability of Gaussian local search is \(1 - \zeta(t)\), then a probability is randomly generated within \([0, 1]\), if the probability is less than \(\zeta(t)\), the new individual is produced randomly, otherwise, the Gaussian local search is used to generate the new individual. Here, the Gaussian local search is described as follows:

\[
x_i = x_i + N(0, \zeta(t)) \quad (20)
\]

The detailed SADI is described in Algorithm 1.

S2 The definition and characteristics of the dynamic benchmark functions

In this section, the definition and characteristic of the ten benchmark functions F1-F11 that are used to test the performance of MOEA-OSD/SRS is shown in Table 1.

<table>
<thead>
<tr>
<th>Benchmark functions</th>
<th>Feasible region</th>
<th>Objectives, PS and PF</th>
<th>Type</th>
</tr>
</thead>
</table>

Table 1: Dynamic benchmark functions
<table>
<thead>
<tr>
<th></th>
<th>$X_I = [x_1] \in [0, 1]$</th>
<th>$X_H = [x_2, ..., x_n] \in [-1, 1]$</th>
<th>where $n = 10$</th>
<th>$f_1(x, t) = x_1$, $f_2(x, t) = g(1 - (\frac{t}{g})^H)$, $g = 1 + 9 \sum_{x_i \in X_H} x_i^2$, $H = 1.25 + 0.75 \sin(0.5\pi t)$.</th>
<th>Type III</th>
</tr>
</thead>
<tbody>
<tr>
<td>F2</td>
<td>$X_I = [x_1] \in [0, 1]$</td>
<td>$X_H = [x_2, ..., x_n] \in [-1, 1]$</td>
<td>where $n = 10$</td>
<td>$f_1(x, t) = x_1$, $f_2(x, t) = g(1 - (\frac{t}{g})^H)$, $g = 1 + \sum_{x_i \in X_H} (x_i - G)^2$, $G = \sin(0.5\pi t)$, $H = 1.25 + 0.75 \sin(0.5\pi t)$.</td>
<td>Type II</td>
</tr>
<tr>
<td>F3</td>
<td>$x_i \in [0, 1]^n$</td>
<td>where $n = 10$</td>
<td>$f_1(x, t) = x_1$, $f_2(x, t) = g(1 - \sqrt{\frac{t}{g}})$, $g = 1 + \sum_{x_i \in X_H} (x_i - G(t))^2$, $G =</td>
<td>\sin(0.5\pi t)</td>
<td>$, $r = \cup (1, 2, ..., n)$</td>
</tr>
<tr>
<td>F4</td>
<td>$X_I = [x_1] \in [0, 1]$</td>
<td>$X_H = [x_2, ..., x_n] \in [-1, 1]$</td>
<td>where $n = 10$</td>
<td>$f_1(x, t) = x_1$, $f_2(x, t) = g(1 - \sqrt{\frac{t}{g}})$, $g = 1 + \sum_{x_i \in X_H} (x_i - G)^2$, $G = \sin(0.5\pi t)$.</td>
<td>Type I</td>
</tr>
<tr>
<td>F5</td>
<td>$X_I = [x_1] \in [0, 1]$</td>
<td>$X_H = [x_2, ..., x_6] \in [-1, 1]$</td>
<td>$X_{III} = [x_7, ..., x_n] \in [-1, 1]$</td>
<td>where $n = 13$</td>
<td>$f_1(x, t) = x_1$, $f_2(x, t) = g \times h$, $g = 1 + \sum_{x_i \in X_H} x_i^2$, $h = 1 - (f_1/g)^2 \left(\frac{H(t) + \sum_{x_i \in X_{III}} (x_i - H(t)/4)^2}{H(t)}\right)$, $H(t) = 2\sin(0.5\pi(t - 1))$.</td>
</tr>
<tr>
<td>F6</td>
<td>$X_I = [x_1] \in [0, 1]$</td>
<td>$X_H = [x_2, ..., x_n] \in [-1, 1]$</td>
<td>where $n = 10$</td>
<td>$f_1(x, t) = x_1^2$, $f_2(x, t) = g(1 - \sqrt{\frac{t}{g}})$, $g = 1 + G + \sum_{x_i \in X_H} (x_i - G)^2$, $G =</td>
<td>\sin(0.5\pi t)</td>
</tr>
<tr>
<td>F7</td>
<td>$x_i \in [0, 1]^n$</td>
<td>where $n = 10$</td>
<td>$f_1(x, t) = (1 + g) \cdot \cos(0.5\pi \cdot x_2) \cdot \cos(0.5\pi \cdot x_1)$, $f_2(x, t) = (1 + g) \cdot \cos(0.5\pi \cdot x_2) \cdot \sin(0.5\pi \cdot x_1)$, $f_3(x, t) = (1 + g) \cdot \sin(0.5\pi \cdot x_2)$, $g = \sum_{i=3}^{n} (x_i - G(t))^2$, $G(t) =</td>
<td>\sin(0.5\pi \cdot t)</td>
<td>$.</td>
</tr>
</tbody>
</table>

A Self-adaptive Response Strategy for MOEA-OSD (MOEA-OSD/SRS)

**Evolutionary Computation**  Volume x, Number x  5
S3.1 Inverted generational distance

Objective problems and 1156 for the three-objective problems.

**Inverted generational distance (IGD)** is a widely used indicator to evaluate the performance of MOEAs because it can evaluate both convergence and diversity, which is defined as follows:

\[
IGD_t(PF_t^*, PF_t) = \frac{\sum_{v \in PF_t^*} d(v, PF_t)}{|PF_t^*|}
\]

where \(PF_t\) is the \(PF\) obtained by an MOEA at time instant \(t\), and \(PF_t^*\) is the true \(PF\), \(v\) is an individual in \(PF_t^*\), \(d(v, PF_t)\) is the Euclidean distance between the individual \(v\) and its closest neighbor in \(PF_t\), \(|PF_t^*|\) is the size of \(PF_t^*\).

<table>
<thead>
<tr>
<th>(F8)</th>
<th>(x_i \in [0, 1]^n) where (n = 10)</th>
<th>(f_1(x, t) = (1 + g) \cdot \cos(0.5 \pi \cdot y_2) \cdot \cos(0.5 \pi \cdot y_1))</th>
<th>Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F9)</td>
<td>(X_I = [x_1] \in [0, 1]) (X_H = [x_2, ..., x_n] \in [-1, 1]) where (n = 10)</td>
<td>(f_1(x, t) = (1 + g)(x_1 + A_t \sin(W_t \pi x_1))), (f_2(x, t) = (1 + g)(1 - x_1 + A_t \sin(W_t \pi x_1))), (g = \sum_{x_i \in X_H} (x_i - G)^2, G = \sin(0.5 \pi t))</td>
<td>Type II</td>
</tr>
<tr>
<td>(F10)</td>
<td>(X_I = [x_1] \in [0, 1]) (X_H = [x_2, ..., x_n] \in [-1, 1]) where (n = 10)</td>
<td>(f_1(x, t) = (1 + g)(x_1 + A_t \sin(W_t \pi x_1))), (f_2(x, t) = (1 + g)(1 - x_1 + A_t \sin(W_t \pi x_1))), (g = \sum_{x_i \in X_H} (x_i)^2)</td>
<td>Type III</td>
</tr>
<tr>
<td>(F11)</td>
<td>(x_i \in [0, 5]^n) where (n = 10)</td>
<td>(f_1(x, t) =</td>
<td>x_1 - a</td>
</tr>
</tbody>
</table>

It should point that, for the true \(PF\), the number of sample points is 501 for the two-objective problems and 1156 for the three-objective problems.

**S3 Performance Metric**

**S3.1 Inverted generational distance**

Inverted generational distance (IGD) is a widely used indicator to evaluate the performance of MOEAs because it can evaluate both convergence and diversity, which is defined as follows:

\[
IGD_t(PF_t^*, PF_t) = \frac{\sum_{v \in PF_t^*} d(v, PF_t)}{|PF_t^*|}
\]

where \(PF_t\) is the PF obtained by an MOEA at time instant \(t\), and \(PF_t^*\) is the true \(PF\), \(v\) is an individual in \(PF_t^*\), \(d(v, PF_t)\) is the Euclidean distance between the individual \(v\) and its closest neighbor in \(PF_t\), \(|PF_t^*|\) is the size of \(PF_t^*\).
Algorithm 1: self-adaptive diversity introduction (SADI)

**Input**: population $POP$;  
**Output**: population $POP$;

1. Estimating the extent of environmental change according to Eq. (18);
2. Calculate diversity introduction ratio $\zeta(t)$ according to Eq. (19);
3. for ($i = 0; i < \zeta(t) \times \text{popsize}; i++$) do
   4. Randomly select an individual $x$ from population $POP$;
   5. $p = \text{rand}(0, 1)$;
   6. if $p < \zeta(t)$ then
      7. Using random initialization to generate a new individual $x$ to replace the previous $x$;
   8. else
      9. using Gaussian local search $N(0, \zeta(t))$ on $x$;
   10. end
4. end

If the environment changes $T_{\text{max}}$ times, the average $IGD$ of the whole process, i.e. $\overline{IGD}$ is defined as follows:

$$\overline{IGD} = \frac{\sum_{t=1}^{T_{\text{max}}} IGD_t}{T_{\text{max}}}$$  \hspace{1cm} (22)

$\overline{IGD}$ reflects the degree of proximity between the obtained $PF$ and the true $PF$, so the smaller the value of $\overline{IGD}$ is, the better the performance of the algorithm.

**S3.2 Spacing**

Schott developed this metric to measure the evenness of the solutions in $PF$ obtained by an algorithm, and the expression is as follows:

$$S_t = \sqrt{\frac{1}{|PF_t| - 1} \sum_{i=1}^{|PF_t|} (d_i - \overline{d})^2}$$  \hspace{1cm} (23)

$$d_i = \min_{k=1,...,|PF_t|, k \neq i} \left( \sum_{j=1}^{m} |PF_t(i, j) - PF_t(k, j)| \right)$$  \hspace{1cm} (24)

where $d_i$ is the Euclidean distance between the $i$-th solution in $PF_t$ and its nearest solution in $PF_t$, and $\overline{d}$ is the average value of $d_i$.

If the environment changes $T_{\text{max}}$ times, the average $S$ of the whole process, i.e. $\overline{S}$ is defined as follows:

$$\overline{S} = \frac{\sum_{t=1}^{T_{\text{max}}} S_t}{T_{\text{max}}}$$  \hspace{1cm} (25)

The smaller the value of $\overline{S}$ is, the more even the solutions in $PF$ distribute.

**S3.3 Hypervolume**

The hypervolume (HV) measures the area or volume covered by the obtained $PF$, which is calculated as follows:

$$HV_t = \text{volume}(\bigcup_{i=1}^{|PF_t|} v_i)$$  \hspace{1cm} (26)
where $PF_t$ is the $PF$ obtained by the algorithm at time $t$, and $v_i$ is the hypervolume formed by the reference point and $i$-th individual. The reference point for the computation of hypervolume is $(z_1^* \cdot 1.1, z_2^* \cdot 1.1, \ldots, z_m^* \cdot 1.1)$, where $z_j^*$ is the maximum value of the $j$-th objective of the true $PF$ and $m$ is the number of objectives.

If the environment changes $T_{\text{max}}$ times, the average $HV$ of the whole process, i.e. $\overline{HV}$ is defined as follows:

$$\overline{HV} = \frac{\sum_{t=1}^{T_{\text{max}}} HV_t}{T_{\text{max}}}$$  \hspace{1cm} (27)

The larger $\overline{HV}$, the better the performance of the algorithm.

S4  
**Comparison between SRS and Other Six Effective Response Strategies**

S4.1  
**Parameter Settings**

Most parameter settings are based on the recommendations in the original references and the details are presented in Table 2. The following parameters are set to be the same in the seven compared algorithms: (1) Population size: $N = 100$; (2) The number of the individuals selected to detect environmental change, which is set to be 5% of the population size; (3) Differential crossover probability: $CR=0.5$; (4) Differential scale factor: $F=0.5$; (5) Gaussian mutation probability: $p_m = 1/n$. Justifications of the parameter settings will be given in Section S7.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Parameter Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOEA-OSD/RDI</td>
<td>Initial the population when environment changes: 20%</td>
</tr>
<tr>
<td>MOEA-OSD/MDI</td>
<td>Mutate the population when environment changes: 40%</td>
</tr>
<tr>
<td>MOEA-OSD/LPS</td>
<td>/</td>
</tr>
<tr>
<td>MOEA-OSD/FFPS</td>
<td>The AR(p) model order is $p = 3$; The length of time series: $M = 23$; The probability of prediction model: $P = 0.9$; Predicted points: $3(m + 1)$;</td>
</tr>
<tr>
<td>MOEA-OSD/PPS</td>
<td>The AR(p) model order is $p = 3$; The length of time series: $M = 23$;</td>
</tr>
<tr>
<td>MOEA-OSD/SADI</td>
<td>/</td>
</tr>
<tr>
<td>MOEA-OSD/SRS</td>
<td>/</td>
</tr>
</tbody>
</table>

S4.2  
**Experimental Results of $\overline{S}$ and $\overline{HV}$**

In the terms of $\overline{S}$, it can be seen from Table 3, SRS performs better than the best of the first five compared response strategies on 6 instances and performs similarly on the rest 27 instances, according to the Wilcoxon rank-sum test.

As for $\overline{HV}$, as shown in Table 4, SRS is superior on 9 instances and performs comparably on the rest 24 instances compared with the best of the first five response strategies, based on the Wilcoxon rank-sum test.

All the above results confirm that SRS can indeed select the most suited response strategy for different benchmark functions, thereby being capable of solving DMOPs subject to unknown environmental changes.

In addition, the 9th column of Tables 3-4 also present the results of $\overline{S}$ and $\overline{HV}$ obtained by MOEA-OSD/SADI, respectively. Based on these results, we can conclude that MOEA-OSD/SRS has a better performance than MOEA-OSD/SADI on all instances.

S4.3  
**Comparison of the computational efficiency of the seven algorithms**

To compare the computational efficiency of the seven algorithms, Table 5 lists the average running time of 20 independent runs of all algorithms on all benchmark functions when $(\tau_T, n_T)$ is set to (10,10). All algorithms are implemented in MATLAB 2016a and run on the same
Table 3: The statistical results of $\bar{S}$ for different algorithms. "†", "§" and "≈" denote the performance of MOEA-OSD/SRS is better than, worse than, or comparable with that of the best of the first five algorithms

<table>
<thead>
<tr>
<th>functions $\langle \tau^*, n \rangle$</th>
<th>Statistic</th>
<th>MOEA-OSD /RD</th>
<th>MOEA-OSD /MD</th>
<th>MOEA-OSD /PS</th>
<th>MOEA-OSD /PS</th>
<th>MOEA-OSD /SADI</th>
<th>MOEA-OSD /SRS</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20,10)</td>
<td>Mean</td>
<td>0.0062</td>
<td>0.0059</td>
<td>0.0071</td>
<td>0.0059</td>
<td>0.0062</td>
<td>0.0080</td>
<td>0.0058</td>
</tr>
<tr>
<td>std</td>
<td>0.0072</td>
<td>0.0063</td>
<td>0.0082</td>
<td>0.0085</td>
<td>0.0099</td>
<td>0.0101</td>
<td>0.0104</td>
<td>0.2453</td>
</tr>
<tr>
<td>(15,10)</td>
<td>Mean</td>
<td>1.39E-04</td>
<td>1.30E-04</td>
<td>9.88E-05</td>
<td>1.53E-04</td>
<td>1.64E-04</td>
<td>5.1E-04</td>
<td>1.14E-04</td>
</tr>
<tr>
<td>std</td>
<td>0.0099</td>
<td>0.0078</td>
<td>0.0103</td>
<td>0.0109</td>
<td>0.0084</td>
<td>0.0138</td>
<td>0.0075</td>
<td>0.2453</td>
</tr>
<tr>
<td>(10,10)</td>
<td>Mean</td>
<td>6.59E-05</td>
<td>5.38E-05</td>
<td>3.94E-05</td>
<td>1.53E-04</td>
<td>1.58E-04</td>
<td>6.0E-04</td>
<td>2.0E-04</td>
</tr>
<tr>
<td>std</td>
<td>0.0084</td>
<td>0.0086</td>
<td>0.0076</td>
<td>0.0078</td>
<td>0.0070</td>
<td>0.0086</td>
<td>0.0069</td>
<td>0.2453</td>
</tr>
<tr>
<td>(20,10)</td>
<td>Mean</td>
<td>1.24E-04</td>
<td>8.18E-05</td>
<td>5.24E-05</td>
<td>1.01E-04</td>
<td>8.16E-05</td>
<td>5.1E-04</td>
<td>5.8E-05</td>
</tr>
<tr>
<td>std</td>
<td>0.0093</td>
<td>0.0076</td>
<td>0.0088</td>
<td>0.0078</td>
<td>0.0078</td>
<td>0.0095</td>
<td>0.0016</td>
<td>0.6985</td>
</tr>
<tr>
<td>(15,10)</td>
<td>Mean</td>
<td>5.26E-05</td>
<td>3.62E-05</td>
<td>2.13E-04</td>
<td>9.26E-04</td>
<td>9.7E-04</td>
<td>2.7E-04</td>
<td>1.0E-04</td>
</tr>
<tr>
<td>std</td>
<td>0.0071</td>
<td>0.0075</td>
<td>0.0076</td>
<td>0.0080</td>
<td>0.0070</td>
<td>0.0086</td>
<td>0.0069</td>
<td>0.6985</td>
</tr>
</tbody>
</table>

Evolutionary Computation Volume x, Number x
Table 4: The statistical results of $H^V$ for different algorithms. "∗", "∗∗" and "∗∗∗" denote the performance of MOEA-OSD is better than, worse than, or comparable with that of the best of the first five algorithms.

<table>
<thead>
<tr>
<th>Functions</th>
<th>Statistic</th>
<th>MOEA-OSD</th>
<th>MOEA-OSD</th>
<th>MOEA-OSD</th>
<th>MOEA-OSD</th>
<th>MOEA-OSD</th>
<th>MOEA-OSD</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(τ↑, nτ↓)</td>
<td></td>
<td>RDI</td>
<td>MDI</td>
<td>FPS</td>
<td>SADI</td>
<td>SRS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F1</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.6547</td>
<td>0.6549</td>
<td>0.6505</td>
<td>0.6549</td>
<td>0.6528</td>
<td>0.1600</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.8524</td>
<td>0.6537</td>
<td>0.6465</td>
<td>0.6537</td>
<td>0.6504</td>
<td>0.6262</td>
<td>0.6580</td>
</tr>
<tr>
<td>F2</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.6466</td>
<td>0.6472</td>
<td>0.6511</td>
<td>0.6483</td>
<td>0.6502</td>
<td>0.6449</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.5194-05</td>
<td>2.35E-04</td>
<td>1.79E-05</td>
<td>9.01E-05</td>
<td>1.73E-04</td>
<td>2.29E-04</td>
<td>6.73E-05</td>
</tr>
<tr>
<td>F3</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.6398</td>
<td>0.6416</td>
<td>0.6490</td>
<td>0.6432</td>
<td>0.6472</td>
<td>0.6355</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.571E-04</td>
<td>2.90E-05</td>
<td>9.20E-05</td>
<td>2.72E-04</td>
<td>9.13E-04</td>
<td>1.84E-04</td>
<td></td>
</tr>
<tr>
<td>F4</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.6198</td>
<td>0.6261</td>
<td>0.6428</td>
<td>0.6312</td>
<td>0.6307</td>
<td>0.6045</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.831E-04</td>
<td>9.43E-05</td>
<td>6.59E-04</td>
<td>6.89E-04</td>
<td>2.05E-04</td>
<td>0.0544</td>
<td>0.2143</td>
</tr>
<tr>
<td>F5</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.6858</td>
<td>0.6890</td>
<td>0.7065</td>
<td>0.7012</td>
<td>0.8232</td>
<td>0.8614</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.8588</td>
<td>0.8626</td>
<td>0.8646</td>
<td>0.8665</td>
<td>0.8666</td>
<td>0.8677</td>
<td>0.8691</td>
</tr>
<tr>
<td>F6</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.7086</td>
<td>0.7126</td>
<td>0.7136</td>
<td>0.7102</td>
<td>0.6374</td>
<td>0.6778</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.8279</td>
<td>0.8360</td>
<td>0.8387</td>
<td>0.8450</td>
<td>0.6818</td>
<td>0.0013</td>
<td>1.22E-04</td>
</tr>
<tr>
<td>F7</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.802E-04</td>
<td>0.808E-04</td>
<td>0.8098</td>
<td>0.8099</td>
<td>0.8099</td>
<td>0.8099</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.826E-04</td>
<td>0.827E-04</td>
<td>0.827E-04</td>
<td>0.827E-04</td>
<td>0.827E-04</td>
<td>0.827E-04</td>
<td>0.827E-04</td>
</tr>
<tr>
<td>F8</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.769E-04</td>
<td>0.769E-04</td>
<td>0.769E-04</td>
<td>0.769E-04</td>
<td>0.769E-04</td>
<td>0.769E-04</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.819E-04</td>
<td>0.819E-04</td>
<td>0.819E-04</td>
<td>0.819E-04</td>
<td>0.819E-04</td>
<td>0.819E-04</td>
<td>0.819E-04</td>
</tr>
<tr>
<td>F9</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.697E-04</td>
<td>0.697E-04</td>
<td>0.697E-04</td>
<td>0.697E-04</td>
<td>0.697E-04</td>
<td>0.697E-04</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.713E-04</td>
<td>0.713E-04</td>
<td>0.713E-04</td>
<td>0.713E-04</td>
<td>0.713E-04</td>
<td>0.713E-04</td>
<td>0.713E-04</td>
</tr>
<tr>
<td>F10</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.689E-04</td>
<td>0.689E-04</td>
<td>0.689E-04</td>
<td>0.689E-04</td>
<td>0.689E-04</td>
<td>0.689E-04</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
</tr>
<tr>
<td>F11</td>
<td>(10,10)</td>
<td>Mean</td>
<td>0.707E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
<td>0.708E-04</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.706E-04</td>
<td>0.706E-04</td>
<td>0.706E-04</td>
<td>0.706E-04</td>
<td>0.706E-04</td>
<td>0.706E-04</td>
<td>0.706E-04</td>
</tr>
</tbody>
</table>
A Self-adaptive Response Strategy for MOEA-OSD (MOEA-OSD/SRS)

machine with four-core (3.2-gigahertz) CPU, 8.0 gigabytes RAM, and Windows 7 operation system. We can observe that the computational complexity of SRS is slightly higher than that of other response strategies. On average, MOEA-OSD/SADI is the fastest algorithm, followed by MOEA-OSD/RDI, MOEA-OSD/MDI, MOEA-OSD/FPS, MOEA-OSD/LPS and MOEA-OSD/PPS. This is reasonable since SRS needs to implement all five response strategies. When an environmental change occurs, the five response strategies all generate a new population, then SRS selects different ratios of individuals from each population to form a new population to respond to the environmental change.

Table 5: The runtime of each algorithm (s)

<table>
<thead>
<tr>
<th>functions</th>
<th>MOEA-OSD/RDI</th>
<th>MOEA-OSD/MDI</th>
<th>MOEA-OSD/LPS</th>
<th>MOEA-OSD/FPS</th>
<th>MOEA-OSD/PSS</th>
<th>MOEA-OSD/SADI</th>
<th>MOEA-OSD/SRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>29.110(3)</td>
<td>28.189(2)</td>
<td>30.015(4)</td>
<td>30.919(5)</td>
<td>34.476(7)</td>
<td>26.395(1)</td>
<td>34.180(6)</td>
</tr>
<tr>
<td>F2</td>
<td>29.498(3)</td>
<td>29.484(2)</td>
<td>30.295(4)</td>
<td>29.531(5)</td>
<td>32.729(6)</td>
<td>24.789(1)</td>
<td>34.773(7)</td>
</tr>
<tr>
<td>F3</td>
<td>27.597(2)</td>
<td>26.957(1)</td>
<td>27.940(3)</td>
<td>28.517(4)</td>
<td>33.805(6)</td>
<td>28.985(5)</td>
<td>34.757(7)</td>
</tr>
<tr>
<td>F4</td>
<td>29.983(2)</td>
<td>30.685(3)</td>
<td>31.731(5)</td>
<td>30.795(4)</td>
<td>31.918(6)</td>
<td>24.461(1)</td>
<td>35.662(7)</td>
</tr>
<tr>
<td>F5</td>
<td>27.643(2)</td>
<td>27.285(1)</td>
<td>28.267(3)</td>
<td>27.634(4)</td>
<td>29.515(6)</td>
<td>27.706(5)</td>
<td>34.117(7)</td>
</tr>
<tr>
<td>F6</td>
<td>36.239(2)</td>
<td>37.066(4)</td>
<td>37.347(5)</td>
<td>37.799(6)</td>
<td>36.780(3)</td>
<td>25.210(1)</td>
<td>54.195(7)</td>
</tr>
<tr>
<td>F7</td>
<td>45.053(3)</td>
<td>45.896(4)</td>
<td>47.643(5)</td>
<td>44.117(2)</td>
<td>48.189(6)</td>
<td>41.606(1)</td>
<td>66.956(7)</td>
</tr>
<tr>
<td>F8</td>
<td>28.970(2)</td>
<td>29.281(3)</td>
<td>29.656(4)</td>
<td>28.975(5)</td>
<td>30.607(6)</td>
<td>24.461(1)</td>
<td>36.957(7)</td>
</tr>
<tr>
<td>F9</td>
<td>36.255(3)</td>
<td>35.002(2)</td>
<td>36.112(4)</td>
<td>36.250(5)</td>
<td>36.202(6)</td>
<td>34.210(1)</td>
<td>40.607(7)</td>
</tr>
<tr>
<td>Mean-rank</td>
<td>2.6</td>
<td>2.3</td>
<td>4.1</td>
<td>4.4</td>
<td>5.8</td>
<td>1.9</td>
<td>6.9</td>
</tr>
</tbody>
</table>

S5 Comparison between MOEA-OSD/SRS and Other Seven DMOAs

S5.1 Parameter Settings

Most parameters are set based on those recommended in the original references, and the details are presented in Table 6. For a fair comparison, 5% solutions of the population are picked to detect the environmental change for all the six algorithms.

S5.2 Comparison the PF obtained by eight algorithms

To visualize the performance of the algorithms under comparison, this section presents the comparative results of the PFs obtained by the eight algorithms. In Figs.1-2, the red stars represent the PF obtained by the compared algorithms, while the black lines represent the true PF.

Fig.1 presents the obtained PF of the eight algorithms on F1 at \(t = 49, 59, 62, 64, 67, 69\), when \((\tau_T, n_T)\) is set as \((10, 10)\). It can be seen that MOEA-OSD/SRS, SGEA and Immune-GDE3 can converge to the true PF completely, and each of the above three algorithms has a good distribution. The distribution of the PFs obtained by the other four algorithms is not very good, and some solutions obtained by DNSGAII-A, FPS-RM-MEDA, PPS-RM-MEDA and MOEA/D have not converged to the true PF.

Fig.2 shows the obtained PF of the eight algorithms on F4 at \(t = 9, 19, 29, 39, 49, 59, 62, 79, 89, 99\) when \((\tau_T, n_T)\) is set to \((10, 10)\). Because F4 is a Type I DMOP, i.e., the PF of F4 does not change over time, both the obtained PF and the true PF are moved by \((f_1 + t/20, f_2 + t/20)\) to make the experimental results clearer. It is clear that the solutions obtained by MOEA-OSD/SRS have converged to the true PF fully, and show a good distribution. SGEA and Immune-GDE3 also can converge to the real PF along over time, too, although the distribution of SGEA and Immune-GDE3 is not as good as that of MOEA-OSD/SRS, and some solutions they obtained have not converged. DNSGAII-A, DNSGAII-B, FPS-RM-MEDA, PPS-RM-MEDA and MOEA/D obtain a few solutions only, i.e.
Ruochen Liu, Jianxia Li, Yaochu Jin, and Licheng Jiao

Table 6: Parameter settings of the six algorithms

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Parameter Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNSGA-II-A</td>
<td>Population size: ( N = 100 ); SBX Crossover probability: ( p_c = 1 ); Cross distribution index: 15; Polynomial mutation probability: ( p_m = 1/n ); Mutation distribution index: 20; Mutate the population when environment changes: 20% (DNSGA-II-A); Initial the population when environment changes: 20% (DNSGA-II-B);</td>
</tr>
<tr>
<td>DNSGA-II-B</td>
<td>SBX Crossover probability: ( p_c = 1 ); Cross distribution index: 15; Polynomial mutation probability: ( p_m = 1/n ); Mutation distribution index: 20;</td>
</tr>
<tr>
<td>FPS-RM-MEDA</td>
<td>Population size: ( N = 100 ); The AR(p) model: ( p = 3 ); The length of time series: ( M = 23 ); The number of the predicted points: ( 3(m + 1) ); The probability of prediction model: ( P = 0.9 );</td>
</tr>
<tr>
<td>SGEA</td>
<td>Population size: ( N = 100 ); SBX Crossover probability: ( p_c = 1 ); Crossover distribution index: 20; Polynomial mutation probability: ( p_m = 1/n ); Mutation distribution index: 20;</td>
</tr>
<tr>
<td>MOEA/D</td>
<td>Population size: ( N = 100 ); The number of the neighbor weight vectors of each weight vectors: ( T = 20 ); SBX Crossover probability: ( p_c = 1 ); Crossover distribution index: 20; Polynomial mutation probability: ( p_m = 1/n ); Mutation distribution index: 20;</td>
</tr>
<tr>
<td>Immune-GDE3</td>
<td>Differential crossover probability: ( CR=0.8 ); Differential scale factor: ( F=0.5 ); Mutation rates: 0.9; Memory size: 100;</td>
</tr>
<tr>
<td>MOEA-OSD/SRS</td>
<td>Population size: ( N = 100 ); Differential crossover probability: ( CR=0.5 ); Differential scale factor: ( F=0.5 ); Gaussian mutation probability: ( p_m = 1/n );</td>
</tr>
</tbody>
</table>

they can only converge to the real PF only at some environments.

S5.3 Comparison of the computational efficiency of the eight algorithms

In order to compare the computational efficiency of the eight algorithms, Table 7 summarizes the average runtime of 20 independent runs of the compared algorithms on all benchmark functions when \( (\tau_T, n_T) \) is set as \((10,10)\). We can note that MOEA-OSD/SRS is the fastest algorithm among the eight compared algorithms, whereas DNSGA-II-B is the most time-consuming algorithm overall, as indicated by the mean-rank of runtime over all 10 problems.

Table 7: The runtime of each algorithm (s)

<table>
<thead>
<tr>
<th>functions</th>
<th>DNSGA-II-A</th>
<th>DNSGA-II-B</th>
<th>FPS-RM-MEDA</th>
<th>PPS-RM-MEDA</th>
<th>SGEA</th>
<th>MOEA/D</th>
<th>Immune-GDE3</th>
<th>MOEA-OSD/SRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>218.716(7)</td>
<td>220.235(8)</td>
<td>56.192(4)</td>
<td>40.794(2)</td>
<td>79.729(6)</td>
<td>55.443(3)</td>
<td>70.520(5)</td>
<td>34.180(1)</td>
</tr>
<tr>
<td>F2</td>
<td>219.576(7)</td>
<td>219.900(8)</td>
<td>56.176(4)</td>
<td>46.067(2)</td>
<td>74.800(6)</td>
<td>50.076(3)</td>
<td>68.113(5)</td>
<td>34.773(1)</td>
</tr>
<tr>
<td>F3</td>
<td>198.000(7)</td>
<td>199.449(8)</td>
<td>47.736(3)</td>
<td>44.070(2)</td>
<td>64.582(5)</td>
<td>48.080(4)</td>
<td>64.852(6)</td>
<td>34.757(1)</td>
</tr>
<tr>
<td>F4</td>
<td>207.901(7)</td>
<td>211.235(8)</td>
<td>53.836(4)</td>
<td>42.183(2)</td>
<td>68.050(6)</td>
<td>46.847(3)</td>
<td>66.771(5)</td>
<td>35.662(1)</td>
</tr>
<tr>
<td>F5</td>
<td>206.323(8)</td>
<td>205.366(7)</td>
<td>48.033(4)</td>
<td>45.053(2)</td>
<td>66.825(6)</td>
<td>46.831(3)</td>
<td>66.321(5)</td>
<td>33.540(1)</td>
</tr>
<tr>
<td>F6</td>
<td>203.423(7)</td>
<td>204.731(8)</td>
<td>53.836(4)</td>
<td>42.183(2)</td>
<td>68.050(6)</td>
<td>46.847(3)</td>
<td>66.771(5)</td>
<td>35.662(1)</td>
</tr>
<tr>
<td>F7</td>
<td>223.304(7)</td>
<td>226.926(8)</td>
<td>50.123(2)</td>
<td>41.559(1)</td>
<td>82.001(5)</td>
<td>50.029(3)</td>
<td>82.320(6)</td>
<td>54.195(4)</td>
</tr>
<tr>
<td>F8</td>
<td>202.829(7)</td>
<td>203.436(8)</td>
<td>55.355(2)</td>
<td>54.944(1)</td>
<td>79.990(5)</td>
<td>62.322(3)</td>
<td>81.543(6)</td>
<td>66.996(4)</td>
</tr>
<tr>
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<td>225.464(8)</td>
<td>224.848(7)</td>
<td>62.369(4)</td>
<td>45.950(3)</td>
<td>72.335(6)</td>
<td>49.648(2)</td>
<td>68.326(5)</td>
<td>36.957(1)</td>
</tr>
<tr>
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<td>216.276(7)</td>
<td>214.299(8)</td>
<td>54.632(4)</td>
<td>39.640(2)</td>
<td>77.879(6)</td>
<td>50.263(3)</td>
<td>69.687(5)</td>
<td>36.255(1)</td>
</tr>
<tr>
<td>F11</td>
<td>231.214(8)</td>
<td>231.082(7)</td>
<td>65.078(3)</td>
<td>67.829(4)</td>
<td>80.202(6)</td>
<td>54.685(2)</td>
<td>72.562(5)</td>
<td>40.607(1)</td>
</tr>
<tr>
<td>Mean-rank</td>
<td>7.4</td>
<td>7.6</td>
<td>3.3</td>
<td>2.3</td>
<td>5.6</td>
<td>2.8</td>
<td>5.4</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Figure 1: The PF of F4 when $(\tau_T, n_T)$ is set as $(10, 10)$ at $t = 9, 19, 29, 39, 49, 59, 9, 69, 79, 89, 99$. 

Evolutionary Computation  Volume x, Number x
Figure 2: The PF of F1 when $({\tau_T}, {n_T})$ is set as $(10, 10)$ at $t = 49, 59, 62, 64, 67, 69$. 
S6  Comparison between MOEA-OSD and six multi-objective optimization algorithms

In this section, we introduce a comparison to indicate the better performance of MOEA-OSD, the comparison is between MOEA-OSD and other six multi-objective optimization algorithms, which is NSGA-II (Deb et al., 2007), SPEAII (Zitzler et al., 2001), NNIA (Gong et al., 2008), MOEA/D (Zhang and Li, 2007), SMS-EMOA (Beume et al., 2007) and NSGA-II/DE (Li and Zhang, 2009).

S6.1  Benchmark Functions

The ten MOPs are described in this section. The five ZDT problems are developed by Zitzler et al. (Zitzler et al., 2000), it includes ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6. The following five 3-objective DTLZ problems are designed by Deb et al. (Deb et al., 2002), they are DTLZ1, DTLZ2, DTLZ3, DTLZ4 and DTLZ6. Their definition and characteristics is shown in Table 8.

Table 8: Dynamic benchmark functions

<table>
<thead>
<tr>
<th>Benchmark functions</th>
<th>Feasible region</th>
<th>Objectives, PS and PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZDT1 $x_i \in [0,1], i = 1, 2, ..., n$ where $n = 30$</td>
<td>$f_1(x) = x_1$, $f_2(x) = g(x)(1 - \sqrt{f_1(x)}/g(x))$</td>
<td>$g(x) = 1 + 9 \sum_{i=2}^{n} x_i/(n-1)$</td>
</tr>
<tr>
<td>ZDT2 $x_i \in [0,1], i = 1, 2, ..., n$ where $n = 30$</td>
<td>$f_1(x) = x_1$, $f_2(x) = g(x)(1 - (f_1(x)/g(x))^2)$</td>
<td>$g(x) = 1 + 9 \sum_{i=2}^{n} x_i/(n-1)$</td>
</tr>
<tr>
<td>ZDT3 $x_i \in [0,1], i = 1, 2, ..., n$ where $n = 30$</td>
<td>$f_1(x) = x_1$, $f_2(x) = g(x)(1 - \sqrt{f_1(x)/g(x)}) - f_1(x)/g(x)\sin(10\pi x_1)$</td>
<td>$g(x) = 1 + 9 \sum_{i=2}^{n} x_i/(n-1)$</td>
</tr>
<tr>
<td>ZDT4 $x_1 \in [0,1]$ $x_i \in [-5,5], i = 1, 2, ..., n$ where $n = 10$</td>
<td>$f_1(x) = x_1$, $f_2(x) = g(x)(1 - \sqrt{f_1(x)/g(x)})$</td>
<td>$g(x) = 1 + 10(n-1) + \sum_{i=2}^{n} (x_i^2 - 10\cos(4\pi x_i))$</td>
</tr>
<tr>
<td>ZDT6 $x_i \in [0,1], i = 1, 2, ..., n$ where $n = 10$</td>
<td>$f_1(x) = 1 - \exp(-4x_1)\sin^6(6\pi x_1)$, $f_2(x) = g(x)(1 - (f_1(x)/g(x))^2)$</td>
<td>$g(x) = 1 + 9 \sum_{i=2}^{n} x_i/(n-1)^0.25$</td>
</tr>
<tr>
<td>DTLZ1 $x_i \in [0,1], i = 1, 2, ..., n$ where $n = 7$</td>
<td>$f_1(x) = 0.5x_1x_2\cdots x_{k-1} (1 + g(x))$</td>
<td>$f_2(x) = 0.5x_1x_2\cdots (1 - x_{k-1}) (1 + g(x))$</td>
</tr>
<tr>
<td></td>
<td>$\cdots$</td>
<td>$f_{k-1}(x) = 0.5x_1 (1 - x_2) (1 + g(x))$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_k(x) = 0.5 (1 - x_1) (1 + g(x))$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$g(x) = 100 \left(\sqrt{x_1} + \sum_{i \in z_k} (x_i - 0.5)^2 - \cos (20\pi (x_1 - 0.5))\right)$</td>
</tr>
<tr>
<td>DTLZ2 $x_i \in [0,1], i = 1, 2, ..., n$ where $n = 12$</td>
<td>$f_1(x) = (1 + g(x_k)) \cos (0.5x_1\pi) \cdots \cos (0.5x_{k-1}\pi)$</td>
<td>$f_2(x) = (1 + g(x_k)) \cos (0.5x_1\pi) \cdots \sin (0.5x_{k-1}\pi)$</td>
</tr>
<tr>
<td></td>
<td>$\cdots$</td>
<td>$f_k(x) = (1 + g(x_k)) \cos (0.5x_{2}\pi)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_k(x) = (1 + g(x_k)) \sin (0.5x_{2}\pi)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$g(x) = \sum_{x_i \in z_k} (x_i - 0.5)^2$</td>
</tr>
</tbody>
</table>
For DTLZ1 $x_k = 5$, for DTLZ2, DTLZ3 and DTLZ4 $x_k = 10$, for DTLZ6 $x_k = 20$. The PFs of the above ten functions are shown in Fig. 3.

### S6.2 Comparison Algorithms and Parameter Settings

In this section, we introduce a comparison between MOEA-OSD and NSGA-II (Deb et al., 2007), SPEAII (Zitzler et al., 2001), NNIA (Gong et al., 2008), MOEA/D (Zhang and Li, 2007), SMS-EMOA (Beume et al., 2007) and NSGA-II/DE (Li and Zhang, 2009).

And in this section, we present the parameter settings of the seven algorithms and most of the parameter settings are based on original references, the details are presented in Table 9. And for all the six algorithms, the population size is set as 100 for all the benchmark functions, and the number of the iterations is set as: 100 for two-objective function, 200 for three-objective function. We also run 20 independent experiments on each benchmark function of each algorithm.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>NSGA-II</th>
<th>SPEAII</th>
<th>NNIA</th>
<th>MOEA/D</th>
<th>SMS-EMOA</th>
<th>NSGA-II/DE</th>
<th>MOEA-OSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_c$</td>
<td>0.8</td>
<td>0.8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$p_m$</td>
<td>1/m</td>
<td>1/m</td>
<td>1/m</td>
<td>1/m</td>
<td>1/m</td>
<td>1/m</td>
<td>1/m</td>
</tr>
</tbody>
</table>

### S6.3 Experimental Results

This section discusses the statistical results of $IGD$ over 20 runs for the seven algorithms are shown in Table 10.
A Self-adaptive Response Strategy for MOEA-OSD (MOEA-OSD/SRS)

Figure 3: The PFS of the ten benchmark functions.
In Table 10, the bold represents the best algorithm of the seven algorithms, i.e. the smaller the value, the better performance the algorithm. The mean_rank represents the rank of the mean of IGD over 20 runs.

From Table 10, we could find that MODA-OSD outperforms the other six algorithms on average, as indicated by the rank of the mean of IGD over 20 runs of all the 10 problems, followed are SMS-EMOA, MOEA/D, SPEAII, NNIA, NSGA-II/DE and NSGA-II, maybe the performance of those six algorithms is good and bad for different benchmark functions.

As to the stability of all the seven algorithms, relatively speaking, MOEA-OSD is the most stable algorithm on average, as indicated by the rank of the standard deviation of IGD over 20 runs of all the 10 problems.

In order to compare the computational efficiency of the seven algorithms, Table 11 shows the mean run time of 20 independent runs of all the benchmark functions for all algorithms. All the algorithms are implemented on the MATLAB 2016a environment and run on the same machine with four-core (3.2- gigahertz) CPU, 8.0 gigabytes RAM, and Windows 7 operation system. It is obvious that our algorithm, MOEA-OSD, is the second fastest algorithm among those seven algorithms, SPEAII is the most time-consuming algorithm on average, as indicated by the mean-rank of runtime over all the 10 problems. Combining the above experimental results with the result in Table 11, we could find that MOEA-OSD has the best performance with the smaller runtime.

Table 10: The statistical results of IGD for different algorithms

<table>
<thead>
<tr>
<th>Problems</th>
<th>Statistic</th>
<th>NSGA-II</th>
<th>SPEAII</th>
<th>NNIA</th>
<th>MOEA/D</th>
<th>SMS-EMOA</th>
<th>NSGA/DE</th>
<th>MOEA-OSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZDT1</td>
<td>Mean</td>
<td>0.0047(3)</td>
<td>0.0039(1)</td>
<td>0.0075(6)</td>
<td>0.0051(4)</td>
<td>0.0058(5)</td>
<td>0.0082(7)</td>
<td>0.0042(2)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>7.05E-04(5)</td>
<td>5.43E-04(4)</td>
<td>9.79E-04(7)</td>
<td>3.57E-04(2)</td>
<td>3.26E-04(4)</td>
<td>4.75E-04(4)</td>
<td>1.41E-04(1)</td>
</tr>
<tr>
<td>ZDT2</td>
<td>Mean</td>
<td>0.0060(3)</td>
<td>0.0190(6)</td>
<td>0.1611(7)</td>
<td>0.0043(2)</td>
<td>0.0089(4)</td>
<td>0.0094(5)</td>
<td>0.0041(1)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>5.43E-04(4)</td>
<td>9.79E-04(7)</td>
<td>3.26E-04(2)</td>
<td>3.57E-04(2)</td>
<td>3.26E-04(2)</td>
<td>6.34E-04(4)</td>
<td>1.13E-05(1)</td>
</tr>
<tr>
<td>ZDT3</td>
<td>Mean</td>
<td>0.0119(6)</td>
<td>0.0098(3)</td>
<td>0.0088(2)</td>
<td>0.0294(7)</td>
<td>0.0060(1)</td>
<td>0.0116(5)</td>
<td>0.0106(4)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.0246(5)</td>
<td>0.0110(5)</td>
<td>0.0095(4)</td>
<td>0.0251(7)</td>
<td>3.57E-04(2)</td>
<td>0.0014(3)</td>
<td>0.0106(4)</td>
</tr>
<tr>
<td>ZDT4</td>
<td>Mean</td>
<td>0.4440(5)</td>
<td>0.4231(4)</td>
<td>0.3862(3)</td>
<td>0.0616(2)</td>
<td>0.0036(1)</td>
<td>0.0036(1)</td>
<td>10.7741(7)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.1872(3)</td>
<td>0.2959(4)</td>
<td>0.4374(5)</td>
<td>0.0719(2)</td>
<td>3.57E-04(2)</td>
<td>0.0014(3)</td>
<td>2.1872(6)</td>
</tr>
<tr>
<td>ZDT6</td>
<td>Mean</td>
<td>0.0180(6)</td>
<td>0.0165(5)</td>
<td>0.0154(4)</td>
<td>0.0026(1)</td>
<td>0.0615(7)</td>
<td>0.0037(3)</td>
<td>0.0032(2)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.0036(5)</td>
<td>0.0033(3)</td>
<td>0.0037(6)</td>
<td>0.0034(4)</td>
<td>0.0294(7)</td>
<td>6.94E-04(2)</td>
<td>5.02E-04(1)</td>
</tr>
<tr>
<td>DTLZ1</td>
<td>Mean</td>
<td>0.1966(6)</td>
<td>0.1491(7)</td>
<td>0.2110(5)</td>
<td>0.1124(4)</td>
<td>0.0292(2)</td>
<td>0.0317(3)</td>
<td>0.0207(1)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.1753(5)</td>
<td>0.1442(4)</td>
<td>0.2964(7)</td>
<td>0.2628(6)</td>
<td>1.04E-04(2)</td>
<td>0.0039(3)</td>
<td>5.52E-04(1)</td>
</tr>
<tr>
<td>DTLZ2</td>
<td>Mean</td>
<td>0.0682(4)</td>
<td>0.0542(1)</td>
<td>0.0721(6)</td>
<td>0.065(3)</td>
<td>0.0793(7)</td>
<td>0.0703(5)</td>
<td>0.0558(2)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.0026(4)</td>
<td>3.67E-04(3)</td>
<td>0.0058(6)</td>
<td>7.04E-04(2)</td>
<td>0.0093(7)</td>
<td>0.0082(5)</td>
<td>2.07E-04(1)</td>
</tr>
<tr>
<td>DTLZ3</td>
<td>Mean</td>
<td>12.5089(5)</td>
<td>11.9840(3)</td>
<td>12.1286(4)</td>
<td>5.3498(2)</td>
<td>1.8195(1)</td>
<td>36.0969(7)</td>
<td>14.4841(6)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>5.7590(3)</td>
<td>7.2188(5)</td>
<td>10.1502(6)</td>
<td>4.5907(2)</td>
<td>0.9136(1)</td>
<td>6.8739(4)</td>
<td>12.8712(7)</td>
</tr>
<tr>
<td>DTLZ4</td>
<td>Mean</td>
<td>0.2823(6)</td>
<td>0.2569(5)</td>
<td>0.1131(4)</td>
<td>0.3782(3)</td>
<td>0.0823(3)</td>
<td>0.0736(2)</td>
<td>0.0620(1)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.2126(6)</td>
<td>0.253(5)</td>
<td>0.1926(4)</td>
<td>0.3988(7)</td>
<td>0.0083(3)</td>
<td>0.0024(2)</td>
<td>2.48E-04(1)</td>
</tr>
<tr>
<td>DTLZ6</td>
<td>Mean</td>
<td>0.0777(4)</td>
<td>0.0729(2)</td>
<td>0.0807(5)</td>
<td>0.255(6)</td>
<td>0.0727(1)</td>
<td>0.0786(3)</td>
<td>0.1188(6)</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.0038(3)</td>
<td>0.0649(6)</td>
<td>0.0042(4)</td>
<td>0.1545(7)</td>
<td>2.64E-04(1)</td>
<td>0.0049(5)</td>
<td>0.0322(2)</td>
</tr>
</tbody>
</table>

| mean_rank | 4.8 | 3.7 | 4.6 | 3.9 | 3.2 | 4.7 | 3.1 |
| std_rank  | 4.5 | 4.5 | 5.6 | 4.1 | 3.4 | 3.5 | 2.2 |
The runtime of each algorithm is shown in Table 11:

<table>
<thead>
<tr>
<th>Problems</th>
<th>NSGA-II</th>
<th>SPEAII</th>
<th>NNIA</th>
<th>MOEA/D</th>
<th>SMS-EMOA</th>
<th>NSGAII/DE</th>
<th>MOEA-OSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZDT1</td>
<td>24.757(6)</td>
<td>41.387(7)</td>
<td>1.513(1)</td>
<td>6.458(5)</td>
<td>4.4148(4)</td>
<td>3.806(3)</td>
<td>2.090(2)</td>
</tr>
<tr>
<td>ZDT2</td>
<td>24.960(6)</td>
<td>40.670(7)</td>
<td>1.170(1)</td>
<td>4.758(5)</td>
<td>4.2744(4)</td>
<td>3.916(3)</td>
<td>2.527(2)</td>
</tr>
<tr>
<td>ZDT3</td>
<td>23.962(6)</td>
<td>40.420(7)</td>
<td>1.435(1)</td>
<td>5.928(5)</td>
<td>4.3992(4)</td>
<td>3.775(3)</td>
<td>2.324(2)</td>
</tr>
<tr>
<td>ZDT4</td>
<td>25.225(6)</td>
<td>38.376(7)</td>
<td>1.404(1)</td>
<td>5.423(5)</td>
<td>4.3056(3)</td>
<td>8.440(5)</td>
<td>3.931(2)</td>
</tr>
<tr>
<td>ZDT6</td>
<td>25.147(6)</td>
<td>37.890(7)</td>
<td>1.263(1)</td>
<td>5.423(4)</td>
<td>4.2588(3)</td>
<td>5.509(5)</td>
<td>2.340(2)</td>
</tr>
<tr>
<td>DTLZ1</td>
<td>47.990(5)</td>
<td>75.988(6)</td>
<td>2.748(1)</td>
<td>9.254(3)</td>
<td>163.0445(7)</td>
<td>11.076(4)</td>
<td>4.493(2)</td>
</tr>
<tr>
<td>DTLZ2</td>
<td>48.501(5)</td>
<td>96.362(6)</td>
<td>2.278(1)</td>
<td>10.281(4)</td>
<td>240.4128(7)</td>
<td>4.290(2)</td>
<td>5.647(3)</td>
</tr>
<tr>
<td>DTLZ3</td>
<td>47.549(6)</td>
<td>76.472(7)</td>
<td>2.465(1)</td>
<td>9.610(3)</td>
<td>46.6555(5)</td>
<td>11.170(4)</td>
<td>4.586(2)</td>
</tr>
<tr>
<td>DTLZ4</td>
<td>47.003(5)</td>
<td>82.213(6)</td>
<td>3.026(1)</td>
<td>9.391(4)</td>
<td>238.9586(7)</td>
<td>3.853(2)</td>
<td>5.226(3)</td>
</tr>
<tr>
<td>DTLZ6</td>
<td>43.346(5)</td>
<td>96.253(6)</td>
<td>2.512(1)</td>
<td>9.672(4)</td>
<td>239.0510(7)</td>
<td>6.088(3)</td>
<td>5.741(2)</td>
</tr>
<tr>
<td>Mean-rank</td>
<td>5.6</td>
<td>6.6</td>
<td>1</td>
<td>4.1</td>
<td>5.1</td>
<td>3.4</td>
<td>2.2</td>
</tr>
</tbody>
</table>

The comparison between MOEA-OSD and other six state-of-the-art multi-objective optimization algorithms, i.e. NSGA-II, SPEAII, NNIA, MOEA/D, NSGA-II/DE and SMS-EMOA indicate that MOEA-OSD has the best performance with the smaller runtime, relatively speaking.

S7 Sensitivity Analysis

We discuss the influence of some parameters on the performance of MOEA-OSD/SRS, including the severity of the environmental changes, i.e. \( n_T \), the proportion of diversity introduction in RDI and MDI, differential crossover probability (CR), differential scale factor (F), and Gaussian mutation probability.

S7.1 Influence of change severity \( n_T \)

Like \( \tau_T \), change severity \( n_T \) is also an important parameter in DMOPs, which can also affect the performance of the algorithm. To examine the effect of \( n_T \) on the algorithms’ performance, experiments were carried out on F1, F4 and F7 with \( \tau_T \) being fixed to 10, and \( n_T \) set to 5, 10, and 20, which represent severe, moderate, and slight environmental changes, respectively.

Experimental results of the eight comparison algorithms in terms of \( \text{IGD} \), \( S \) and \( HV \) are given in Tables 12-14, respectively. It can be observed from the tables that all the algorithms are sensitive to \( n_T \), as can be seen from the improvement of the metrics when the value of \( n_T \) increases. MOEA-OSD/SRS is able to exhibit impressive performance in terms of \( \text{IGD} \) and \( HV \) on all of the instances under different severity levels. Regarding \( S \), MOEAD-OSD/SRS is superior to other compared algorithms on F1, and SGEA performs the best on F4 and F7.

S7.2 Influence of proportion of diversity introduction in RDI and MDI

The proposed SRS integrates five commonly used response strategies, namely, RDI, MDI, LPS, FPS and PPS. In those five response strategies, all the parameters are set based on the original references. However, the original reference shows that, RDI has better performance when proportion of diversity introduction is 20%-70%, and MDI works the best with a 20%-70% of proportion of diversity introduction. This section examines the proportion of diversity introduction on algorithms’ performance, by conducting experiments on F1 and F4.

Fig.4 presents the \( \text{IGD} \) values obtained by MOEA-OSD/SRS on F1 and F4 under different proportions of diversity introduction when \( (\tau_T, n_T) \) is set as (10, 10). It is clear that the propor-
<table>
<thead>
<tr>
<th>Functions</th>
<th>Statistic</th>
<th>(τ&lt;sub&gt;T&lt;/sub&gt;, n&lt;sub&gt;T&lt;/sub&gt;)</th>
<th>DNSGA-I/-A</th>
<th>DNSGA-I/-B</th>
<th>FPS -RM</th>
<th>FPS -RM</th>
<th>SGEA</th>
<th>MOEA/D</th>
<th>Immune GDE3</th>
<th>MOEA-OSD</th>
<th>MOEA-/SRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Mean</td>
<td>(10,5)</td>
<td>0.0970 †</td>
<td>0.0857 †</td>
<td>0.0217 †</td>
<td>0.0245 †</td>
<td>0.0168</td>
<td>0.0271</td>
<td>0.0235 †</td>
<td>0.0083</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10,10)</td>
<td>0.0307 †</td>
<td>0.0244 †</td>
<td>0.0253 †</td>
<td>0.0156</td>
<td>0.0264</td>
<td>0.0200</td>
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Table 14: The statistical results of $HV$ for different algorithms with different values of $n_T$

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Figure 4: The $IGD$ of F1 and F4 under different proportion of diversity introduction when $(\tau_T, n_T)$ is set as (10, 10).

S7.3 Influence of differential scale factor, differential crossover probability and Gaussian mutation probability

Differential scale factor (F), differential crossover probability (CR) and Gaussian mutation probability ($p_m$) are all the important parameters of MOEA-OSD/SRS. Here we examine the influence of these parameters on the performance.

Figs. 5-7, respectively, plot the $IGD$ value obtained by MOEA-OSD/SRS on F1 and F4 as F, CR and $p_m$ changes. It can be seen from Fig. 5 that differential scale factor has a great effect on the performance of MOEA-OSD/SRS, and MOEA-OSD/SRS performs better when F is set as [0.2-0.6]. In this work, F is set as 0.5.

From Fig. 6, we can see the different settings of CR also influence the performance of MOEA-OSD/SRS. Generally speaking, MOEA-OSD/SRS works well with CR being set in [0.4-0.5].
Figure 5: the $\overline{IGD}$ on F1 and F4 with the change of F.

It can be found from Fig.7 that, the smaller the $p_m$, the better the performance of MOEA-OSD/SRS. In our experiments, $p_m$ is set to $1/n$ (n is dimension of the decision vector, for F1 and F4, $1/n=0.1$).

References


A Self-adaptive Response Strategy for MOEA-OSD (MOEA-OSD/SRS)

Figure 6: the IGD on F1 and F4 with the change of CR.

Figure 7: the IGD on F1 and F4 with the change of $p_m$. 