Online Appendix

This document contains a set of appendixes to accompany "Are Supply Shocks Contractionary? Evidence from Utilization-Adjusted TFP Data" by Julio Garín, Robert Lester, and Eric Sims. This document is intended for online publication only.

A Stochastic Peg

As an alternative to what we do in the main text, which is to approximate the effects of a binding zero lower bound with an interest rate peg of deterministic duration, in this Appendix we consider the case in which the duration of the interest rate peg is stochastic. This is the approach taken in the small-scale NK model in Christiano, Eichenbaum and Rebelo (2011), for example. A stochastic peg length has the advantage that it permits clean closed form solutions, which is not the case for the deterministic peg case. A downside of the stochastic peg case is that it can result in counterintuitive "sign flips" in which the effect of a natural rate shock on output flips sign for a sufficiently long expected duration of the peg.

As in the main text, suppose that the current nominal interest rate equals zero, so in deviation terms we have $i_t = 1 - 1/\beta$. With probability $1 - p$, in period $t + 1$ the central bank returns to an inflation target, which implies that $\mathbb{E}_t \, \pi_{t+1} = \mathbb{E}_t \, x_{t+1} = 0$ and $\mathbb{E}_t \, i_{t+1} = \mathbb{E}_t \, r^f_{t+1}$. With probability p, the nominal interest rate in period $t + 1$ remains at zero. The probability of returning to the strict inflation target in any subsequent period, conditional on arriving in that period with the nominal rate still at zero, is fixed at p. We solve for analytic solutions for π_t and x_t using the method of undetermined coefficients. Using the expression mapping a_t into r^f_t from the text, (4), we can write these solutions as:

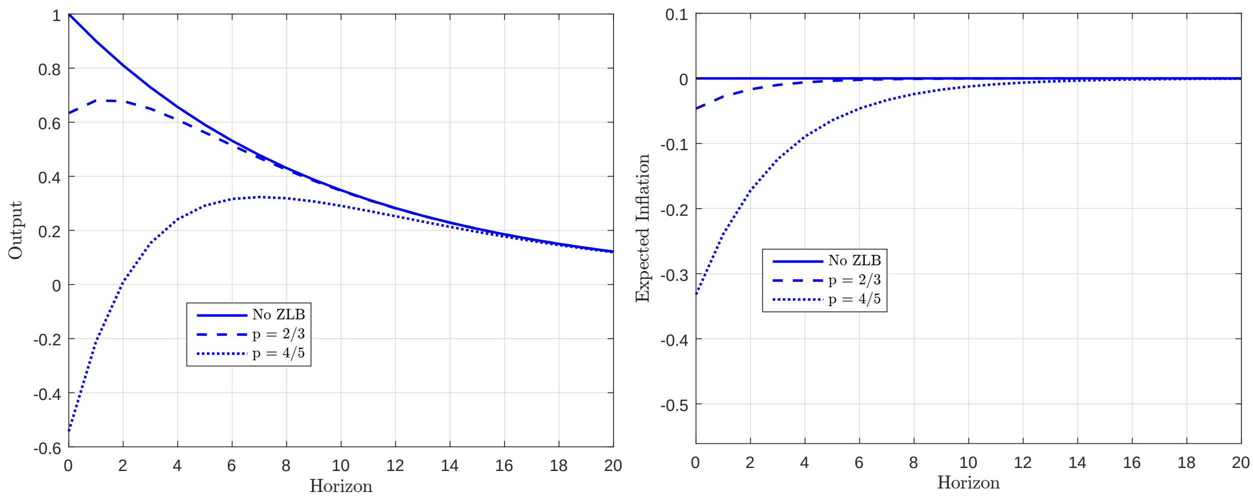
$$\pi_t = \frac{\gamma}{-\sigma(1-\beta p)(1-p) + p\gamma}\left(1 - \frac{1}{\beta}\right) + \frac{\gamma}{\sigma(1-\beta p\rho_a)(1-p\rho_a) - p\rho_a\gamma}\frac{\sigma(1+\chi)(\rho_a - 1)}{\sigma + \chi}a_t \tag{A.1}$$

$$x_t = \frac{1-\beta p}{-\sigma(1-\beta p)(1-p) + p\gamma}\left(1 - \frac{1}{\beta}\right) + \frac{1-\beta p}{\sigma(1-\beta p\rho_a)(1-p\rho_a) - p\rho_a\gamma}\frac{\sigma(1+\chi)(\rho_a - 1)}{\sigma + \chi}a_t. \tag{A.2}$$

Since $x_t = y_t - y^f_t$, and $y^f_t = \frac{1+\chi}{\sigma+\chi}a_t$, this implies that output can be written:

$$y_t = \frac{1-\beta p}{-\sigma(1-\beta p)(1-p) + p\gamma}\left(1 - \frac{1}{\beta}\right) + \left[1 + \frac{\sigma(\rho_a - 1)(1-\beta p)}{\sigma(1-\beta p\rho_a(1-p\rho_a) - p\rho_a\gamma)}\right]\frac{1+\chi}{\sigma+\chi}a_t. \tag{A.3}$$

In Figure A1a we plot impulse responses of output to a productivity shock for two different levels of p: $p \in \{2/3, 4/5\}$. This corresponds to expected durations of three and five quarters, respectively. For point of comparison, we also show the case in which the central bank targets a zero inflation rate in every period, in which case $y_t = y^f_t$. We assume that $\rho_a = 0.90$, $\sigma = 1$, $\chi = 0$, $\beta = 0.99$, and $\phi = 0.75$. Clearly, as the expected duration of the ZLB increases, output expands less on impact in response to a productivity shock. For sufficiently long forecast horizons the responses are not affected much by the value of p, because in expectation the economy will have likely exited the ZLB. Differently than the deterministic peg case, the response under a stochastic peg only asymptotically approaches the flexible price responses, whereas in the deterministic peg the responses lie on top of one another after the peg period.



(a) Output Response

(b) Expected Inflation Response

Figure A1: Responses to a Productivity Shock as a Function of Duration of ZLB From a Stochastic Peg

Notes: These figures plots the impulse responses of output (left) expected inflation (right) to a one percent increase in productivity for different values of p. The time period is a quarter.

Figure A1b is similar to Figure A1a, except we plot the expected inflation response for different expected durations of the peg. As in the main text, the longer is the expected duration of the peg, the more inflation falls on impact.

As documented in Carlstrom, Fuerst and Paustian (2014), for sufficiently high values of p, the sign of the effect of a productivity shock on output and expected inflation can flip. For the values of the other parameters we have chosen, this sign flip occurs at about $p = 0.83$, or an expected duration of the peg of about six quarters. The sign flips do not occur in the deterministic duration case considered in the text. It is important to reiterate that the sign flip applies to both the output and expected inflation response – if output responds more to the productivity shock at the ZLB, then the expected inflation response is positive at the ZLB, rather than negative.

B Taylor Rule

The IS equation and Phillips Curve (PC) are the same as in the main text:

$$x_t = \mathbb{E}_t\, x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\, \pi_{t+1} - r_t^f\right) \tag{B.1}$$

$$\pi_t = \gamma x_t + \beta\, \mathbb{E}_t\, \pi_{t+1}. \tag{B.2}$$

Outside of the ZLB, the interest rate rule is given by

$$i_t = \phi_\pi \pi_t.$$

The process for the natural rate of interest is the same. We first solve for the policy functions of the output gap and inflation outside the ZLB and then consider the ZLB. Substitute the interest rate rule into Equation B.1:

$$x_t = \mathbb{E}_t\, x_{t+1} - \frac{1}{\sigma}\left(\phi_\pi \pi_t - \mathbb{E}_t\, \pi_{t+1} - r_t^f\right).$$

Guess $x_t = \theta_1 a_t$ and $\pi_t = \theta_2 a_t$. After some algebraic manipulations:

$$\begin{aligned}
x_t &= \frac{(1-\beta\rho)\sigma^{-1}}{(1-\beta\rho)(1-\rho) + \frac{\gamma}{\rho}(\phi_\pi - \rho)} \frac{1+\chi}{\sigma+\chi}(\rho-1)a_t \\
\pi_t &= \frac{\gamma\sigma^{-1}}{(1-\beta\rho)(1-\rho) + \frac{\gamma}{\rho}(\phi_\pi - \rho)} \frac{1+\chi}{\sigma+\chi}(\rho-1)a_t.
\end{aligned}$$

The peg runs for H periods and lifts in period H. This implies that $\mathbb{E}_t\, x_{t+H} = \theta_1 \rho^H a_t$ and $\mathbb{E}_t\, \pi_{t+H} = \theta_2 \rho^H a_t$. Since the interest rate is now constrained at 0, this means $\mathbb{E}_t\, i_{t+h} = 1 - 1/\beta$ for all $h < H$. In period $H-1$ we have:

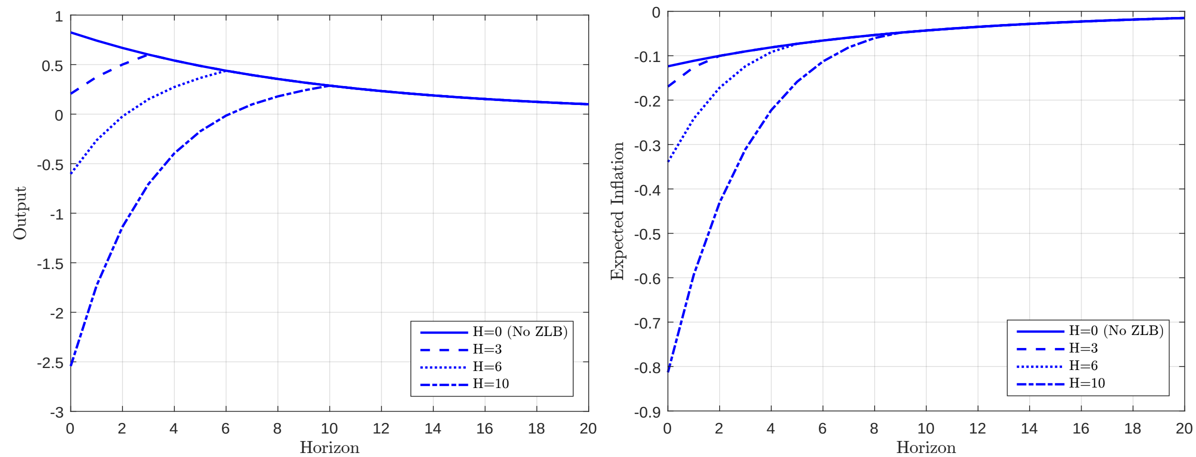
$$\begin{aligned}
\mathbb{E}_t\, x_{t+H-1} &= \mathbb{E}_t x_{t+H} + \frac{1}{\sigma}\left(1 - \frac{1}{\beta} + \mathbb{E}_t\, \pi_{t+H} + \mathbb{E}_t\, r_{t+H-1}^f\right) \\
&= \theta_1 \rho^H a_t + \frac{1}{\sigma}\left(1 - \frac{1}{\beta} + \theta_2 \rho^H a_t + \delta \rho^{H-1} a_t\right)
\end{aligned}$$

where $\delta = \frac{1+\chi}{\chi+\sigma}(\rho - 1)$. Substitute the last expression into Equation B.2:

$$\begin{aligned}
\mathbb{E}_t\, \pi_{t+H-1} &= \gamma\, \mathbb{E}_t\, x_{t+H-1} + \beta\theta_2 \rho^H a_t \\
&= \gamma\left[\theta_1 \rho^H a_t + \frac{1}{\sigma}\left(1 - \frac{1}{\beta} + \theta_2 \rho^H a_t + \delta\rho^{H-1} a_t\right)\right] + \beta\theta_2\rho^H a_t.
\end{aligned}$$

We continue to iterate back to period t.

With the exception of ϕ_π, which is set to 1.5, the rest of the parameterization is identical to the one in the main text. We consider peg lengths of $H \in \{0, 3, 6, 10\}$. Figure B1 presents the results. Note that when $H = 0$ output does not increase by as much as productivity. This is the consequence of price stickiness and it is exactly what inflation targeting avoids, namely non-zero values of the output gap and inflation. Also note that the fall in output for longer peg lengths is significantly bigger than when the unconstrained rule is inflation targeting. Relatedly, output can decline on impact for shorter durations of the peg than in the strict inflation targeting case.



(a) Output Response

(b) Expected Inflation Response

Figure B1: Responses to a Productivity Shock as a Function of Duration of ZLB From a Taylor Rule

Notes: These figures plots the impulse responses of output (left) and expected inflation (right) to a one percent increase in productivity for different durations of a pegged nominal interest rate at zero. $H = 0$ corresponds to the case where the central bank obeys a Taylor Rule.

C Medium Scale Model

In this Appendix we show that the theoretical results derived in Section 2 continue to hold in a medium scale model similar to Smets and Wouters (2007) and Christiano, Eichenbaum and Evans (2005). Specifically, we include: capital accumulation, investment adjustment costs, variable capital accumulation, nominal price and wage rigidities, partial wage and price indexation, and habit formation.

As the model is fairly standard, we only present the first order conditions characterizing the equilibrium of the model:

$$\lambda_t = (C_t - bC_{t-1})^{-1} - \beta b\,\mathbb{E}_t\,(C_{t+1} - bC_t)^{-1} \tag{C.1}$$

$$\lambda_t = \beta(1 + i_t)\,\mathbb{E}_t\,\lambda_{t+1}(1 + \pi_{t+1})^{-1} \tag{C.2}$$

$$\lambda_t R_t = \mu_t\left[\delta_1 + \delta_2(u_t - 1)\right] \tag{C.3}$$

$$\mu_t = \beta\,\mathbb{E}_t\left[\lambda_{t+1}R_{t+1}u_{t+1} + (1 - \delta(u_{t+1}))\mu_{t+1}\right] \tag{C.4}$$

$$\lambda_t = \mu_t\left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right) - \kappa\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right] + \beta\,\mathbb{E}_t\,\mu_{t+1}\kappa\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{C.5}$$

$$w_t^{\#} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{f_{1,t}}{f_{2,t}} \tag{C.6}$$

$$f_{1,t} = \psi\left(\frac{w_t}{w_t^{\#}}\right)^{\epsilon_w(1+\chi)} N_t^{1+\chi} + \beta\theta_w \mathbb{E}_t \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w(1+\chi)} \left(\frac{(1+\pi_t)^{\zeta_w}}{1+\pi_{t+1}}\right)^{-\epsilon_w(1+\chi)} f_{1,t+1} \tag{C.7}$$

$$f_{2,t} = \lambda_t\left(\frac{w_t}{w_t^{\#}}\right)^{\epsilon_w} N_t + \beta\theta_w \mathbb{E}_t \left(\frac{w_{t+1}^{\#}}{w_t^{\#}}\right)^{\epsilon_w} \left(\frac{(1+\pi_t)^{\zeta_w}}{1+\pi_{t+1}}\right)^{1-\epsilon_w} f_{2,t+1} \tag{C.8}$$

$$R_t = \alpha mc_t A_t \widetilde{K}_t^{\alpha-1} N_t^{1-\alpha} \tag{C.9}$$

$$w_t = (1-\alpha) mc_t A_t \widetilde{K}_t^{\alpha} N_t^{-\alpha} \tag{C.10}$$

$$\frac{1+\pi_t^{\#}}{1+\pi_t} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{x_{1,t}}{x_{2,t}} \tag{C.11}$$

$$x_{1,t} = \lambda_t mc_t Y_t + \beta\theta_p \mathbb{E}_t (1+\pi_t)^{-\zeta_p\epsilon_p}(1+\pi_{t+1})^{\epsilon_p} x_{1,t+1} \tag{C.12}$$

$$x_{2,t} = \lambda_t Y_t + \beta\theta_p (1+\pi_t)^{\zeta_p(1-\epsilon_p)} E_t (1+\pi_{t+1})^{\epsilon_p - 1} x_{2,t+1} \tag{C.13}$$

$$K_{t+1} = \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + \left[1 - \delta(u_t)\right] K_t \tag{C.14}$$

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2 \tag{C.15}$$

$$Y_t = C_t + I_t \tag{C.16}$$

$$\widetilde{K}_t = u_t K_t \tag{C.17}$$

$$Y_t v_t^p = A_t \widetilde{K}_t^{\alpha} N_t^{1-\alpha} - F \tag{C.18}$$

$$v_t^p = (1+\pi_t)^{\epsilon_p}\left[(1-\theta_p)(1+\pi_t^{\#})^{-\epsilon_p} + \theta_p(1+\pi_{t-1})^{-\epsilon_p\zeta_p} v_{t-1}^p\right] \tag{C.19}$$

$$(1+\pi_t)^{1-\epsilon_p} = (1-\theta_p)(1+\pi_t^{\#})^{1-\epsilon_p} + \theta_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} \tag{C.20}$$

$$w_t^{1-\epsilon_w} = (1-\theta_w) w_t^{\#,1-\epsilon_w} + \theta_w\left[\frac{(1+\pi_{t-1})^{\zeta_w}}{1+\pi_t} w_{t-1}\right]^{1-\epsilon_w} \tag{C.21}$$

$$i_t = (1-\rho_i)i + \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi(\pi_t - \pi_t^*) + \phi_y(\ln Y_t - \ln Y_{t-1})\right] \tag{C.22}$$

$$1 + r_t = (1+i_t)\mathbb{E}_t(1+\pi_{t+1})^{-1} \tag{C.23}$$

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{A,t}. \tag{C.24}$$

In these equations λ_t is the Lagrange multiplier on the flow budget constraint and μ_t is the Lagrange multiplier on the accumulation equation. (C.1) defines λ_t in terms of the marginal utility of consumption. (C.2) is the Euler equation for bonds, which prices the nominal interest rate, i_t. (C.3) is the first order condition for capital utilization. The optimality condition for the choice of K_{t+1} is (C.4), and the FOC for investment is given by (C.5). Optimal wage-setting for updating households is characterized by (C.6)–(C.8), where $w_t^{\#}$ is the reset real wage, which is common across updating households. Cost-minimization by firms defines the capital-labor ratio and real marginal cost in (C.9)–(C.10). Optimal price-setting for updating firms is characterized by (C.11)–(C.13), where $\pi_t^{\#} = P_t^{\#}/P_{t-1} - 1$ is the reset inflation rate, which is common across updating firms. The aggregate production function is given by (C.18). v_t^p is a measure of price dispersion which can be written recursively as in (C.19). The evolution of aggregate inflation is given by (C.20) and the aggregate real wage evolves according to (C.21). The real interest rate is defined in the Fisher relationship, (C.23). Monetary policy during normal times is characterized by a Taylor rule, given in (C.22). The exogenous process for productivity is given by (C.24), where the non-stochastic level of productivity is normalized to unity. The model is solved by linearization about a zero inflation non-stochastic steady state.

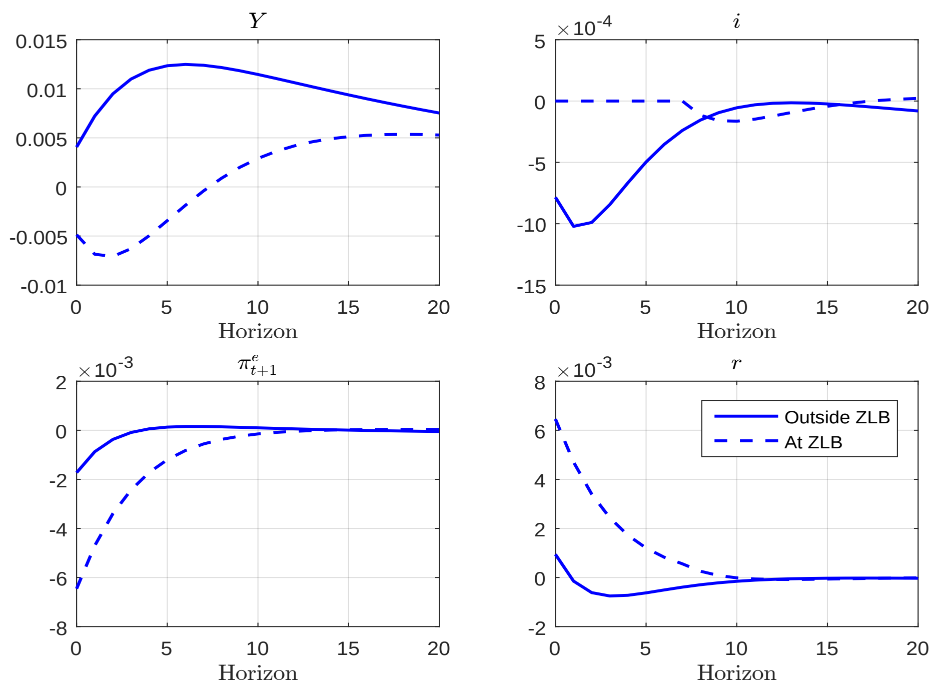


Figure C1: Response to Productivity Shock, With and Without ZLB

Notes: This figure plots impulse responses of output, the nominal interest rate, one period ahead expected inflation, and the real interest rate to a productivity shock in the medium scale DSGE model described in this appendix. The solid blue lines are responses when policy is governed by a Taylor rule, while the dashed blue lines are responses when the nominal interest rate is pegged for eight periods.

To approximate the effects of a binding zero lower bound, we augment the Taylor rule with monetary policy news shocks as in Laseen and Svensson (2011). Then, conditional on a productivity shock, we solve for the values of these news shocks so as to keep the nominal interest rate fixed (in expectation) for a specified period of time. The model is parameterized as follows: $\beta = 0.995$, $\epsilon_p = \epsilon_w = 11$, $\psi = 6$, $\delta_2 = 0.05$, $\alpha = 1/3$, $\chi = 1$, $\theta_w = \theta_p = 0.66$, $\zeta_p = \zeta_w = 0$, $\delta_0 = 0.025$, $b = 0.7$, $\phi_\pi = 1.5$, $\phi_y = 0.2$, $\rho_i = 0.8$, $\kappa = 4$, and $\rho_A = 0.95$. δ_1 is chosen to be consistent with steady state utilization of 1, and F is chosen so that profits are zero in the steady state. We consider a shock to productivity of one percent.

Figure C1 plots impulse responses of output and expected inflation to a positive productivity shock. For completeness we also plot the impulse responses of the nominal rate and the real interest rate. The solid lines are the responses outside the ZLB, the dashed lines are responses when the nominal interest rate is pegged for a duration of eight quarters.

Here, we see exactly the same behavior (qualitatively) as in the model without capital. At the ZLB, output declines, expected inflation falls by more, and the real interest rate rises. Note that the real interest rate rises on impact in both normal times as well as at the ZLB. This is different than the model without capital, where the real interest rate declines after a temporary productivity improvement. Outside of the ZLB, the impact increase in the real interest rate is small and quickly turns negative. What accounts for the difference relative to the textbook model is the presence of capital. A positive productivity shock raises the marginal product of capital, which works to put upward pressure on the real interest rate. In the model with investment adjustment costs, this effect is small and only temporary. Without investment adjustments costs, the impact rise in the real interest rate outside of the ZLB is much stronger and more persistent.

C.1 Permanent Productivity Shocks

Now suppose that exogenous productivity evolves according to a non-stationary stochastic process instead of a mean-reverting process. In particular:

$$\ln g_t = (1-\rho_A)\ln g_A + \rho_A \ln g_{t-1} + \varepsilon_{g,t} \tag{C.25}$$

where $\ln g_t = \ln A_t - \ln A_{t-1}$ and g_A denotes the steady state gross growth rate of productivity. If $\rho_A = 1$, productivity obeys a random walk with drift. If $\rho_A > 0$, then the growth rate of productivity follows a stationary AR(1) process. This specification introduces stochastic trends into the model. Most variables need to be detrended to be rendered stationary. Define $X_t = A_t^{\frac{1}{1-\alpha}}$ as the trend factor. Define $\widehat{H}_t = H_t/X_t$ for generic variable H_t. Exceptions will be λ_t and μ_t, for which the stationary transformations will be $\widehat{\lambda}_t = \lambda_t X_t$ and $\widehat{\mu}_t = \mu_t X_t$. The real and nominal interest rates, the inflation rate, capital utilization, and labor hours will all be stationary without need for transformation.

The full set of detrended equilibrium conditions are presented below:

$$\widehat{\lambda}_t = \left(\widehat{C}_t - b g_{X,t}^{-1}\widehat{C}_{t-1}\right)^{-1} - \beta\,\mathbb{E}_t\, b\left(\widehat{C}_{t+1} g_{X,t+1} - b\widehat{C}_t\right)^{-1} \tag{C.26}$$

$$\widehat{\lambda}_t = \beta(1+i_t)\,\mathbb{E}_t\,\widehat{\lambda}_{t+1} g_{X,t+1}^{-1}(1+\pi_{t+1})^{-1} \tag{C.27}$$

$$\widehat{\lambda}_t = \widehat{\mu}_t\left[\delta_1 + \delta_2(u_t - 1)\right] \tag{C.28}$$

$$\widehat{\mu}_t = \beta\,\mathbb{E}_t\, g_{X,t+1}^{-1}\left[\widehat{\lambda}_{t+1} R_{t+1} u_{t+1} + (1-\delta(u_{t+1}))\widehat{\mu}_{t+1}\right] \tag{C.29}$$

$$\widehat{\lambda}_t = \widehat{\mu}_t \left[1 - \frac{\kappa}{2} \left(\frac{\widehat{I}_t}{\widehat{I}_{t-1}} g_{X,t} - g_X \right) - \kappa \left(\frac{\widehat{I}_t}{\widehat{I}_{t-1}} g_{X,t} - g_X \right) \frac{\widehat{I}_t}{\widehat{I}_{t-1}} g_{X,t} \right] + \beta \, \mathbb{E}_t \, g_{X,t+1}^{-1} \widehat{\mu}_{t+1} \kappa \left(\frac{\widehat{I}_{t+1}}{\widehat{I}_t} g_{X,t+1} - g_X \right) \left(\frac{\widehat{I}_{t+1}}{\widehat{I}_t} g_{X,t+1} \right)^2 \tag{C.30}$$

$$f_{1,t} = \psi \left(\frac{\widehat{w}_t}{\widehat{w}_t^{\#}} \right)^{\epsilon_w(1+\chi)} N_t^{1+\chi} + \beta \theta_w \, \mathbb{E}_t \left(\frac{\widehat{w}_{t+1}^{\#}}{\widehat{w}_t^{\#}} g_{X,t+1} \right)^{\epsilon_w(1+\chi)} \left(\frac{(1+\pi_t)^{\zeta_w}}{1+\pi_{t+1}} \right)^{-\epsilon_w(1+\chi)} f_{1,t+1} \tag{C.31}$$

$$\widehat{f}_{2,t} = \widehat{\lambda}_t \left(\frac{\widehat{w}_t}{\widehat{w}_t^{\#}} \right)^{\epsilon_w} N_t + \beta \theta_w \, \mathbb{E}_t \, g_{X,t+1}^{-1} \left(\frac{\widehat{w}_{t+1}^{\#}}{\widehat{w}_t^{\#}} g_{X,t+1} \right)^{\epsilon_w} \left(\frac{(1+\pi_t)^{\zeta_w}}{1+\pi_{t+1}} \right)^{1-\epsilon_w} \widehat{f}_{2,t+1} \tag{C.32}$$

$$\widehat{w}_t^{\#} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{f_{1,t}}{\widehat{f}_{2,t}} \tag{C.33}$$

$$R_t = \alpha g_{X,t}^{1-\alpha} mc_t \widehat{\widehat{K}}_t^{\alpha} N_t^{1-\alpha} \tag{C.34}$$

$$\widehat{w}_t = (1-\alpha) mc_t g_{X,t}^{-\alpha} \widehat{\widehat{K}}_t^{\alpha} N_t^{-\alpha} \tag{C.35}$$

$$\frac{1+\pi_t^{\#}}{1+\pi_t} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{x_{1,t}}{x_{2,t}} \tag{C.36}$$

$$x_{1,t} = \widehat{\lambda}_t mc_t \widehat{Y}_t + \beta \theta_p \, \mathbb{E}_t (1+\pi_t)^{-\zeta_p \epsilon_p} (1+\pi_{t+1})^{\epsilon_p} x_{1,t+1} \tag{C.37}$$

$$x_{2,t} = \widehat{\lambda}_t \widehat{Y}_t + \beta \theta_p (1+\pi_t)^{\zeta_p(1-\epsilon_p)} \, \mathbb{E}_t (1+\pi_{t+1})^{\epsilon_p - 1} x_{2,t+1} \tag{C.38}$$

$$\widehat{K}_{t+1} = \left[1 - \frac{\kappa}{2} \left(\frac{\widehat{I}_t}{\widehat{I}_{t-1}} g_{X,t} - g_X \right)^2 \right] \widehat{I}_t + [1 - \delta(u_t)] \, g_{X,t}^{-1} \widehat{K}_t \tag{C.39}$$

$$\widehat{Y}_t = \widehat{C}_t + \widehat{I}_t \tag{C.40}$$

$$\widehat{\widehat{K}}_t = u_t \widehat{K}_t \tag{C.41}$$

$$\widehat{Y}_t v_t^p = g_{X,t}^{-\alpha} \widehat{\widehat{K}}_t^{\alpha} N_t^{1-\alpha} - F \tag{C.42}$$

$$v_t^p = (1+\pi_t)^{\epsilon_p} \left[(1-\theta_p)(1+\pi_t^{\#})^{-\epsilon_p} + \theta_p (1+\pi_{t-1})^{-\epsilon_p \zeta_p} v_{t-1}^p \right] \tag{C.43}$$

$$(1+\pi_t)^{1-\epsilon_p} = (1-\theta_p)(1+\pi_t^{\#})^{1-\epsilon_p} + \theta_p (1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} \tag{C.44}$$

$$\widehat{w}_t^{1-\epsilon_w} = (1-\theta_w) \widehat{w}_t^{\#,1-\epsilon_w} + \theta_w \left[\frac{(1+\pi_{t-1})^{\zeta_w}}{1+\pi_t} \widehat{w}_{t-1} g_{X,t}^{-1} \right]^{1-\epsilon_w} \tag{C.45}$$

$$i_t = (1-\rho_i) i + \rho_i i_{t-1} + (1-\rho_i) \left[\phi_\pi (\pi_t - \pi_t^*) + \phi_y (\ln \widehat{Y}_t - \ln \widehat{Y}_{t-1}) \right] \tag{C.46}$$

$$1 + r_t = (1+i_t) \, \mathbb{E}_t (1+\pi_{t+1})^{-1} \tag{C.47}$$

$$\ln g_t = (1-\rho_A) \ln g_A + \rho_A \ln g_{t-1} + \varepsilon_{g,t} \tag{C.48}$$

$$g_{X,t} = g_t^{\frac{1}{1-\alpha}}. \tag{C.49}$$

The equilibrium conditions are the same as in the case with a mean-reverting productivity process, only re-written according to stationary transformations. $g_{X,t}$ is the gross growth rate of the trend factor, and is given by (C.49). With the exception of g_A and ρ_A, the model is parameterized as before.

C.1.1 IRFs in the Permanent Shock Case

We produce IRFs in the permanent shock case for different values of ρ_A. We assume that $g_A = 1$. Our results are not affected by this assumption. First, consider $\rho_A = 0$, so that productivity is an exact random walk. The IRFs of output, the nominal interest rate, the inflation rate, and the real interest rate are shown below in Figure C2. The solid lines depict the responses when policy is governed by the Taylor rule, while the dashed lines represent responses when the nominal interest rate is pegged for eight quarters.

These responses are qualitatively different than the textbook model presented in the text, as well as compared to the responses from the medium scale model when productivity obeys a stationary stochastic process. In particular, output increases by more at the ZLB in comparison to when monetary policy is governed by a Taylor rule. This pattern of response would seem to be consistent with the empirical results presented in Section 3. However, different than our empirical results, in this version of the model expected inflation falls by less at the ZLB compared to normal times. As we discuss in Section 3, empirically it is the joint behavior of output and expected inflation in response to a productivity shock that is difficult to square with the theory, not the response of output in isolation.

Next, consider higher levels of persistence (so that the shock is correlated in growth rates). First, we set $\rho_A = 0.1$. Qualitatively, the responses are similar to the random walk case shown in Figure C2, though the differences between the interest rate peg and Taylor rule case are quantitatively smaller.

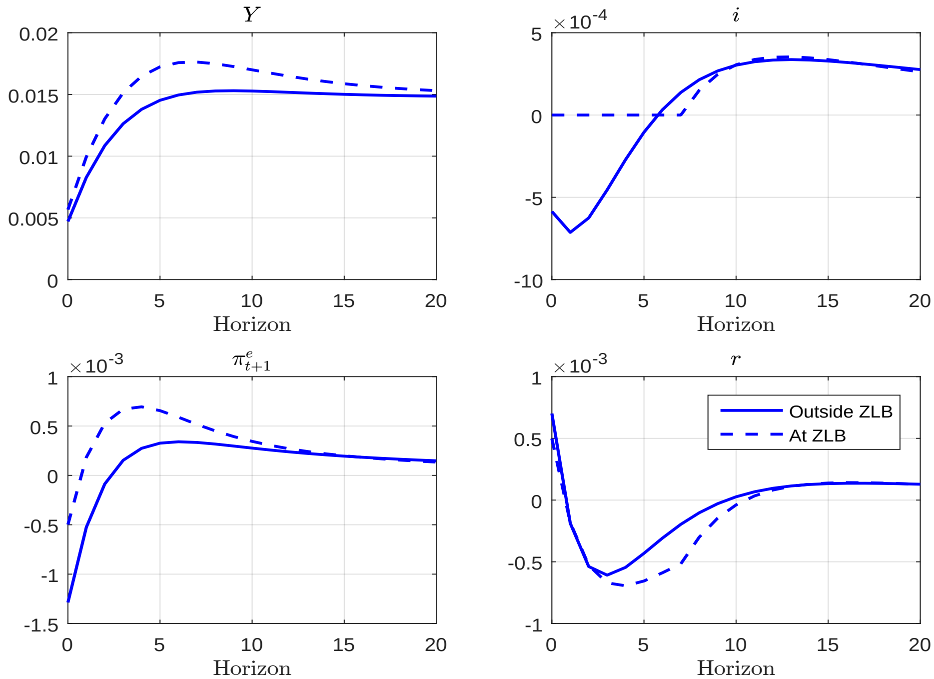


Figure C2: Response to Productivity Shock, With and Without ZLB, Permanent Shock with $\rho_A = 0$

Notes: This figure is similar to Figure C1, but considers the case when productivity follows an exact random walk.

Lastly, we increase the persistence of the productivity process further, setting $\rho_A = 0.4$. The responses are shown in Figure C4. Here we are back in the case where output responds less (and expected inflation falls more). The results are presented in Figure C4. In terms of the impact responses of output and inflation, these responses differ from the random walk case and are more similar to the textbook model and the medium scale model with a stationary productivity shock. Output rises by less, and inflation falls by more, on impact after a positive shock to the growth rate of productivity.

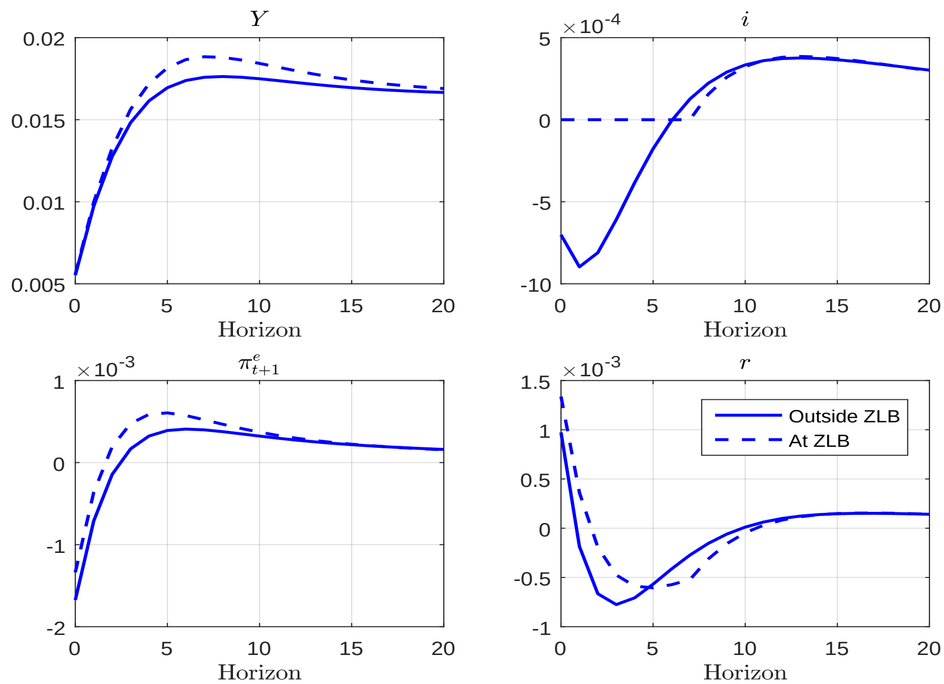


Figure C3: Response to Productivity Shock, With and Without ZLB, Permanent Shock with $\rho_A = 0.1$

Notes: This figure is similar to Figure C2, but considers the case when productivity follows a persistent AR(1) process in the growth rate with $\rho_A = 0.1$.

In summary, the basic logic of the model in Section 2 is preserved when capital and additional real and nominal frictions are added to the model. Output rises by less and expected inflation falls by more at the ZLB than outside of it when the productivity shock is persistent but stationary. When the productivity process is an exact random walk, on the other hand, output and expected inflation increase by more under an interest rate peg than under a Taylor rule. If the productivity process is sufficiently persistent in growth rates, output again rises by less on impact, and expected inflation falls by more, to a positive productivity shock. Though there is not a robust prediction from the medium scale model on the sign of the effect of a binding ZLB on the response of output, what is robust is the joint behavior of output and expected inflation. If expected inflation falls by more at the ZLB, then output increases by less, and vice-versa. This pattern is not consistent with the empirical responses of output and expected inflation to a productivity shock which we identify in the data.

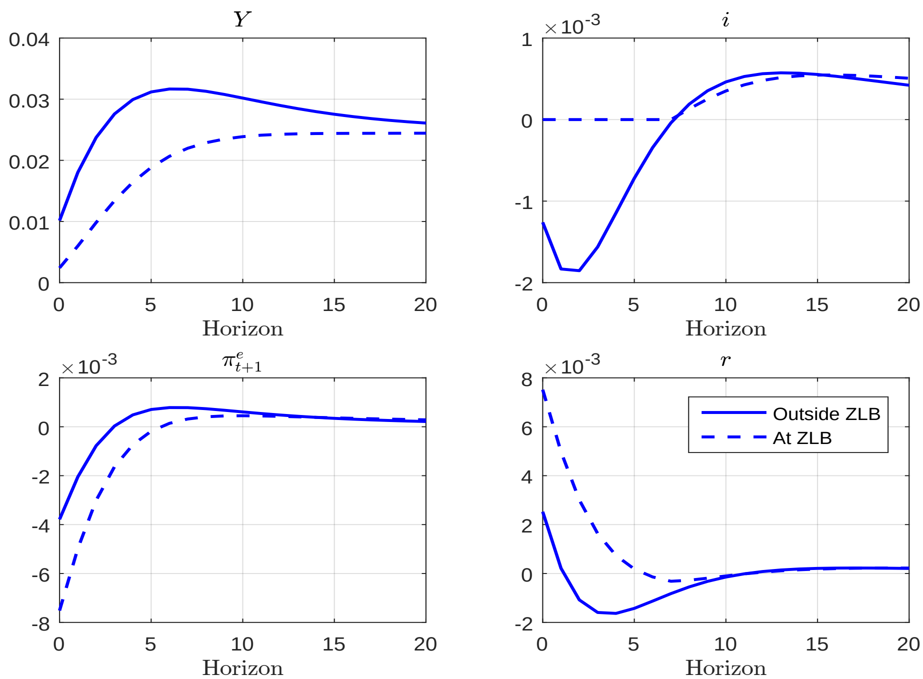


Figure C4: Response to Productivity Shock, With and Without ZLB, Permanent Shock with $\rho_A = 0.4$

Notes: This figure is similar to Figure C2, but considers the case when productivity follows a persistent AR(1) process in the growth rate with $\rho_A = 0.4$.

D Smooth Local Projections

Here we briefly outline the smooth local projections (SLP) methodology of Barnichon and Brownlees (2016). In describing the SLP methodology, we abstract from state-dependence. It is straightforward to modify the model to include state-dependence. Suppose that one is interested in the following local projection:

$$Y_{t+h} = \alpha^h + \beta^h X_t + \sum_{i=1}^{N} \gamma_i^h W_{i,t} + u_{t+h}. \tag{D.1}$$

In (D.1), X_t is the regressor of interest, and β^h measures the estimated impulse response at forecast horizon $h \geq 0$. We sometimes also refer to β^h as the "dynamic multiplier." $W_{i,t}$, for $i = 1, \ldots, N$, are control variables (for example lags of Y_t and X_t). The sample size is T periods, so t runs from $t = 1, \ldots, T$. We consider responses out to some horizon $H \geq 0$.

SLP makes use of penalized B-spline smoothing following Eilers and Marx (1996). Let $\mathbf{B}$ be a $(H+1) \times K$ matrix. We follow Barnichon and Brownlees (2016) in using a cubic B-spline, so $K = 3 + (H+1)$. Let $B_k(h)$, for $k = 1, \ldots, K$ and $h = 0, \ldots, H$, denote the k^{th} column of the h^{th} row of $\mathbf{B}$. As in Barnichon and Brownlees (2016), we only smooth the dynamic multiplier of interest, although it is straightforward to smooth all coefficients. The local projection (D.1) can be approximated as:

$$Y_{t+h} \approx \alpha^h + \sum_{k=1}^{K} b_k B_k(h) X_t + \sum_{i=1}^{N} \gamma_i^h W_{i,t} + u_{t+h}. \tag{D.2}$$

Here, b_k, $k = 1, \ldots, K$, are a set of scalar parameters to be estimated. These parameters are the coefficients on $B_k(h)X_t$ for $k = 1, \ldots, K$. Eilers and Marx (1996) provide Matlab code for computing $\mathbf{B}$, the B-spline base matrix. We follow the details provided in Barnichon and Brownlees (2016).[1] The impulse response coefficient of interest is $\beta^h \approx \sum_{k=1}^{K} b_k B_k(h)$. Once estimates of $\widehat{b}_k$ are obtained, given the B-spline base matrix, it is straightforward to recover the impulse response coefficient.

The B-spline approximation of the local projection, (D.2), can be written in matrix form. We employ the following notation. θ is a vector of parameters:

$$\theta = \begin{pmatrix} b_1 & \ldots & b_k & \alpha^h & \gamma_1^h & \ldots & \gamma_N^h \end{pmatrix}'. \tag{D.4}$$

Let $\widetilde{\mathbf{Y}}$ be a $T(H+1) \times 1$ vector, where the $1+(t-1)(H+1)+h$ entry equals Y_{t+h} for $t = 1, \ldots, T$ and $h = 0, \ldots, H$. Entries which would require observations of Y_j where $j > T$ are left blank. After removing blank entries, define $\mathbf{Y}$ as the $\left(T(H+1) - \frac{H(H+1)}{2}\right) \times 1$ vector of non-blank elements of $\widetilde{\mathbf{Y}}$. If $H = 0$, this is just a vector of Y_t observations running from $t = 1, \ldots, T$. If $H = 1$, then $\mathbf{Y} = \begin{bmatrix} Y_t & Y_{t+1} & Y_{t+1} & Y_{t+2} & \ldots & Y_{t+T-1} & Y_{t+T} & Y_{t+T} \end{bmatrix}'$ and so on for larger values of H.

Similarly, let $\widetilde{\mathbf{X}}$ be a $T(H+1) \times K$ matrix, where the $(1+(t-1)(H+1)+h, k)$ element is $B_k(h)X_t$ for $t = 1, \ldots, T$, $h = 0, \ldots, H$, and $k = 1, \ldots, K$. Entries which would correspond to observations of Y_j with $j > T$ are dropped. The remaining $\left(T(H+1) - \frac{H(H+1)}{2}\right) \times K$ entries are collected into the matrix $\mathbf{X}_\beta$. Let $\widetilde{\mathbf{X}}_\alpha$ be a $T(H+1) \times (H+1)$ matrix. The $(1 + (t-1)(H+1) + h, j+1)$ element is 1 for all $t = 1, \ldots, T$, $h = 0, \ldots, H$, and $j = 0, \ldots, H$ if $j = H$ and zero otherwise. Similarly, elements of this matrix which would correspond to values of the left hand side variable outside of the sample are dropped, leaving $\mathbf{X}_\alpha$ as a $\left(T(H+1) - \frac{H(H+1)}{2}\right) \times (H+1)$ matrix corresponding to the constant in the projection. Finally, let $\mathbf{X}_{\gamma_i}$ be similarly defined as a $\left(T(H+1) - \frac{H(H+1)}{2}\right) \times (H+1)$ matrix corresponding to control variables $i = 1, \ldots N$. Horizontally stacking these matrixes together yields a $\left(T(H+1) - \frac{H(H+1)}{2}\right) \times (K + (H+1)(N+1))$ matrix:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_\beta & \mathbf{X}_\alpha & \mathbf{X}_{\gamma_1} & \ldots \mathbf{X}_{\gamma_N} \end{bmatrix}. \tag{D.5}$$

[1] For example, with $H = 2$, our $\mathbf{B}$ matrix is:

$$\mathbf{B} = \begin{bmatrix} 0.1667 & 0.6667 & 0.1667 & 0 & 0 & 0 \\ 0 & 0.1667 & 0.6667 & 0.1667 & 0 & 0 \\ 0 & 0 & 0.1667 & 0.6667 & 0.1667 & 0 \end{bmatrix}. \tag{D.3}$$

In matrix notation, (D.2) can therefore be written:

$$\mathbf{Y} = \mathbf{X}\theta + \mathbf{u}. \tag{D.6}$$

The $\left(T(H+1) - \frac{H(H+1)}{2}\right) \times 1$ vector $\mathbf{u}$ is a vector of prediction errors.

Following Barnichon and Brownlees (2016) and the smoothing literature, we estimate the parameter vector θ using generalized ridge estimation. This estimator minimizes the penalized residual sum of squares and is given by:

$$\begin{aligned}\widehat{\theta} &= \arg\min_{\theta} \quad (\mathbf{Y} - \mathbf{X}\theta)' (\mathbf{Y} - \mathbf{X}\theta) + \lambda\theta'\mathbf{P}\theta \\ &= (\mathbf{X}'\mathbf{X} + \lambda\mathbf{P})^{-1} \mathbf{X}'\mathbf{Y}. \end{aligned} \tag{D.7}$$

In (D.7) λ is a scalar penalty parameter and $\mathbf{P}$ is a $(K + (H+1)(N+1)) \times (K + (H+1)(N+1))$ penalty matrix. We set $\lambda = 100$, though our results are robust to different values. The penalty matrix $\mathbf{P}$ is chosen in such a way as to allow one to shrink the dynamic multiplier, $\beta^h \approx \sum_{k=1}^{K} b_k B_k(h)$, to a polynomial of given order, r. Let $\mathbf{D}$ be an identity matrix of dimension $K \times K$. Then let $\mathbf{D}_r$ denote the rth difference of $\mathbf{D}$, which is a matrix of dimension $(K - r) \times K$.[2] We then let the first $K \times K$ elements of $\mathbf{P}$ be $\mathbf{D}_r'\mathbf{D}_r$, while all remaining elements are zero. For our work in the paper, we follow Barnichon and Brownlees (2016) in choosing $r = 3$. This has the effect of shrinking the estimated impulse response to a polynomial of order $r - 1 = 2$ to the extent to which λ is large.

Given an estimate of $\widehat{\theta}$ from (D.7), it is straightforward to recover the estimates of the dynamic multipliers, $\widehat{\beta}^h \approx \sum_{k=1}^{K} \widehat{b}_k B_k(h)$. Given a vector of prediction errors, $\widehat{\mathbf{u}} = \mathbf{Y} - \mathbf{X}\widehat{\theta}$, we construct a Newey-West estimator for the variance-covariance matrix of $\widehat{\theta}$. Since the impulse response functions are just linear combinations of the elements of $\widehat{\theta}$, it is straightforward to recover standard errors for these response coefficients.

D.1 Monte Carlo Illustration

To study the suitability of the SLP methodology, we consider a simple Monte Carlo exercise. Suppose that we have a data generating process which follows an exact VAR(1):

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{pmatrix} 0.7 & 0.1 \\ 0.8 & 0.2 \end{pmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{pmatrix} 1 & 0 \\ 0.2 & 1 \end{pmatrix} \begin{bmatrix} \varepsilon_{X,t} \\ \varepsilon_{Y,t} \end{bmatrix}, \quad \varepsilon_{X,t} \sim N(0,1), \ \varepsilon_{Y,t} \sim N(0,1) \tag{D.8}$$

[2] For example, if $r = 3$ and $H = 2$, so that $K = 6$, then $\mathbf{D}_r$ is:

$$\mathbf{D}_r = \begin{bmatrix} -1 & 3 & -3 & 1 & 0 & 0 \\ 0 & -1 & 3 & -3 & 1 & 0 \\ 0 & 0 & -1 & 3 & -3 & 1 \end{bmatrix}$$

In the data generating process we have imposed a recursive structure, wherein the shock to X_t can affect Y_t on impact but not vice-versa.

We generate $N = 1,000$ different data samples with $T = 300$ observations each. The simulations start at initial values of $X_{t-1} = Y_{t-1} = 0$, so we drop the first 100 simulated periods in each simulation to eliminate the influence of initial conditions. This leaves $1,000$ data sets with 200 observations each. On each simulated data set, we seek to estimate the impulse response of Y_t to a shock to $\varepsilon_{X,t}$. We do so by three different methods. In the first, we estimate a VAR(1) on simulated data and impose a Choleski ordering with Y_t ordered second to recover the impulse response. In the second, we estimate a local projection of the form:

$$Y_{t+h} = \alpha^h + \beta^h X_t + \gamma_Y^h Y_{t-1} + \gamma_X^h X_{t-1} + u_{t+h} \tag{D.9}$$

While the local projection in (D.9) looks similar to the second equation of a VAR system, note that we estimate $H + 1$ separate forecasting regressions of this sort. For the third estimation approach, we estimate a smooth local projection of (D.9), using penalty value $\lambda = 100$ and $r = 3$, as we do in the text.

Figure D1 summarizes the results. Solid lines correspond to the true impulse response of Y_t to $\varepsilon_{X,t}$ in the data generating process. Dashed black lines are the responses obtained from estimating a VAR(1) on simulated data. Dotted black lines are the responses estimated from a conventional local projection. Solid blue lines are the responses obtained from estimating a smooth local projection on simulated data.

The upper left plot shows the impulse responses averaged across the $N = 1,000$ different simulations. There is little noticeable difference among the different methodologies for obtaining the impulse response – all do qualitatively well. If anything, the SLP and LP methodologies perform better than the VAR (as evidenced by being less downward-biased at longer forecast horizons).

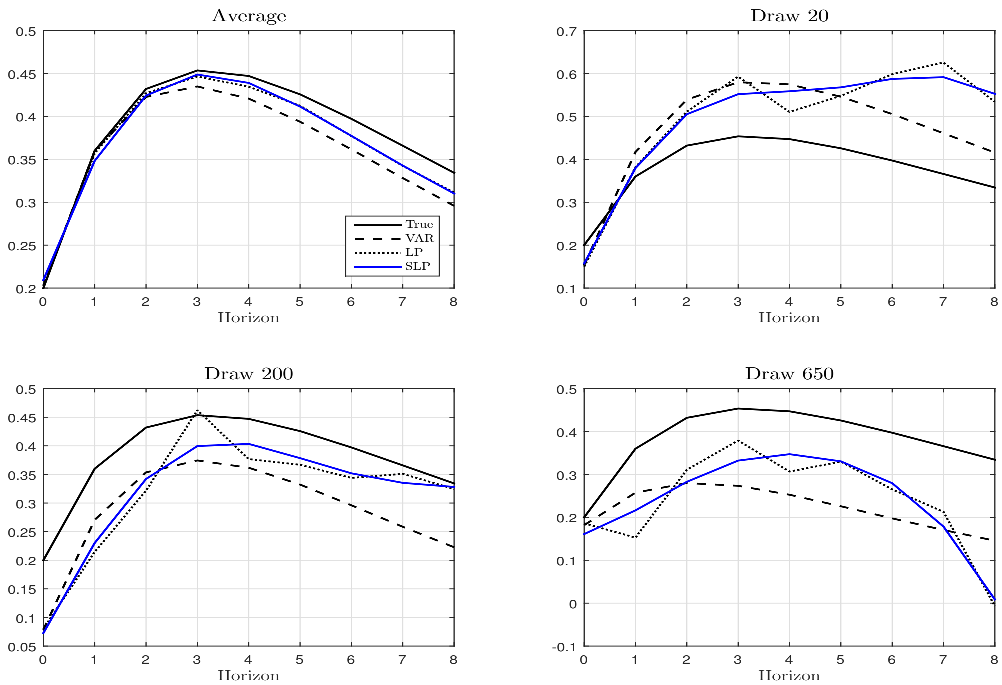


Figure D1: Estimated Impulse Responses From Monte Carlo Exercise

Notes: This figure plots the true response of Y_t to $\varepsilon_{X,t}$ in the data generating process, (D.8). Dashed lines correspond to responses obtained from estimating a VAR(1), dotted lines the responses from a local projection (D.9), and solid blue lines the responses obtained from estimating a smooth local projection version of (D.9). The upper left panel shows the average of estimated responses across 1000 different simulations, while the remaining three plots in the figure correspond to the denoted draw of simulated data.

The remaining three plots in the figure (upper right and the lower row) show estimated responses obtained from particular simulated data sets. In particular, we show responses on the 20th, 200th, and 650th draws of the $N = 1,000$ different simulations. These draws are arbitrarily chosen but serve to illustrate the potential benefits of the SLP methodology. Qualitatively, all the different methodologies capture the true model impulse response fairly well. Nevertheless, on particular draws of data it is evident that the impulse responses obtained via a conventional local projection are quite "choppy." The responses from the SLP are essentially just smoothed versions of those responses.

We next examine the roles played by r and λ. Recall from above that the penalized estimation is designed to push the estimated impulse response function to a polynomial of order r in the limit as λ gets sufficiently large. Figure D2 plots the true impulse response of Y_t to $\varepsilon_{X,t}$ in the model as the solid line. Similarly to Figure D1, there are four panels, with the upper left plotting average responses, the upper right plotting estimated responses from the 20th draw of data, and the bottom row plotting estimated responses from the 200th and 650th draws of data in the simulation. The dashed black, dotted black, and solid blue lines refer, respectively, to the SLP estimate of the impulse response for values of $r \in \{2, 3, 4\}$. These responses are obtained holding λ fixed at 100. Visually, there is not much difference between the SLP responses with different values of r, though one can observe that the estimated responses are somewhat less smooth with larger values of r. This is intuitive – as r is bigger, one is pushing the estimated impulse response to a polynomial of higher order, which allows for more non-linearity.

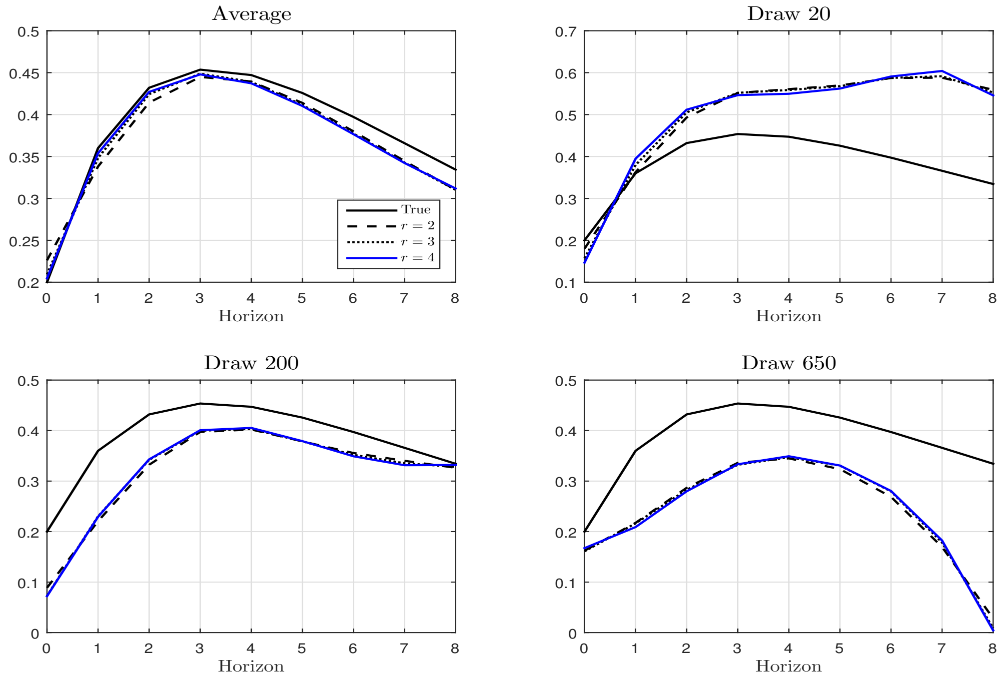


Figure D2: Effect of Varying r

Notes: This figure is constructed similarly to Figure D1, but only plots responses obtained from smooth local projections for different values of r. Solid lines show the true response from the data generating process.

Figure D3 is similar to Figure D2, but instead considers the effects of varying λ holding r fixed at its baseline value of 3. We consider three different values of λ: 1, 100, and 10,000. Here there are more noticeable differences than is evident when varying r. In particular, when λ is lower, the estimated SLP responses are relatively more choppy, whereas when λ is larger the responses are smoother. This again makes sense, as λ measures the penalty for the estimated responses differing from a polynomial of order r.

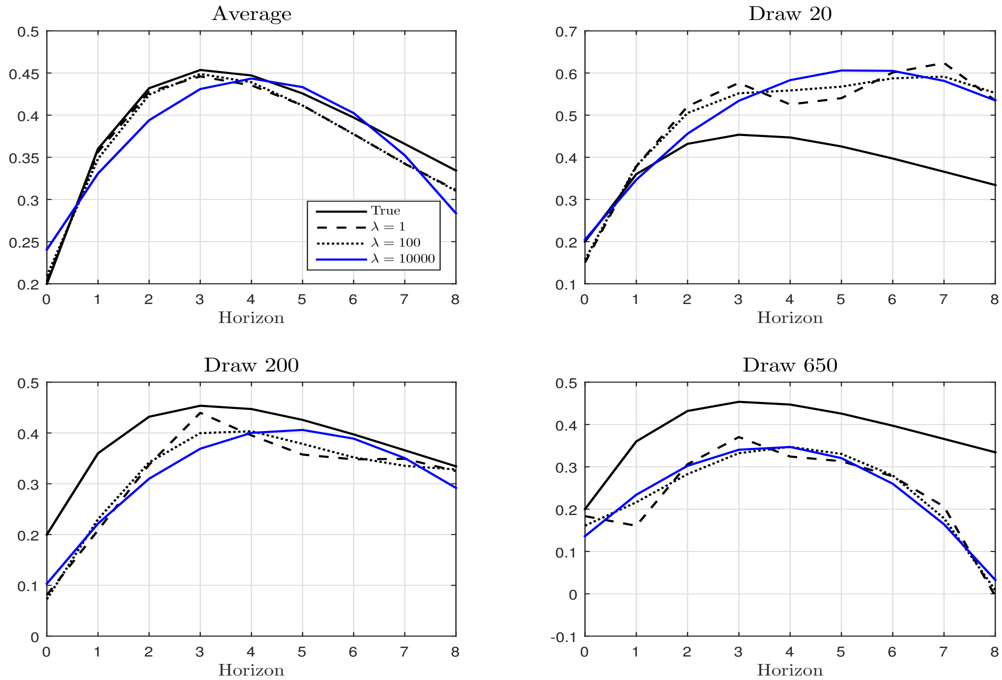


Figure D3: Effect of Varying λ

Notes: This figure is constructed similarly to Figure D2, but only plots responses obtained from smooth local projections for different values of λ. Solid lines show the true response from the data generating process.

E Empirical Results Under Conventional Local Projections

This appendix shows impulse responses and confidence intervals for output and expected inflation obtained under a conventional local projection, as opposed to the smooth local projection on which we focus in the text.

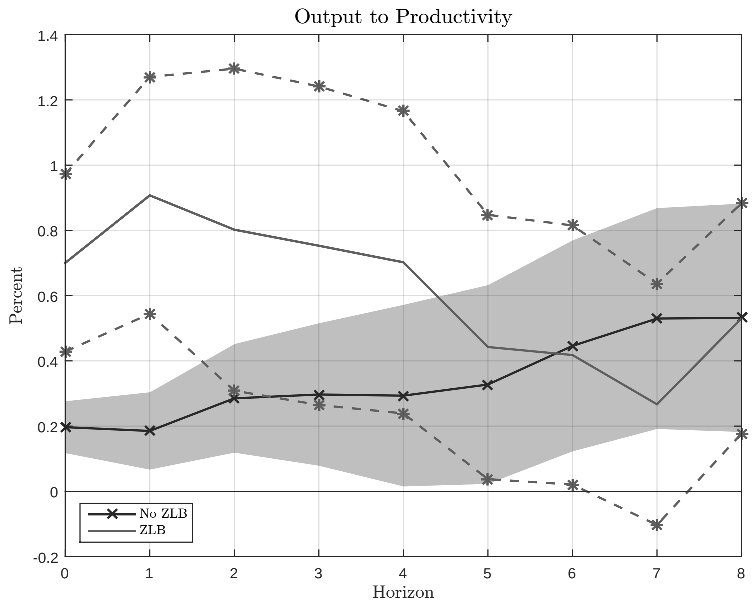


Figure E1: Results from Baseline LP, Output

Notes: This figure shows the estimate impulse response of output to a one unit productivity shock at various horizons. The line with 'x' markers covers the case when the ZLB does not bind (i.e. when $Z_t = 0$). The shaded area bands represent the 90 percent confidence interval about the no ZLB case. The solid line shows responses when the ZLB binds or $Z_t = 1$. Theotted lines with '*' markers represent the 90 percent confidence interval about the ZLB case.

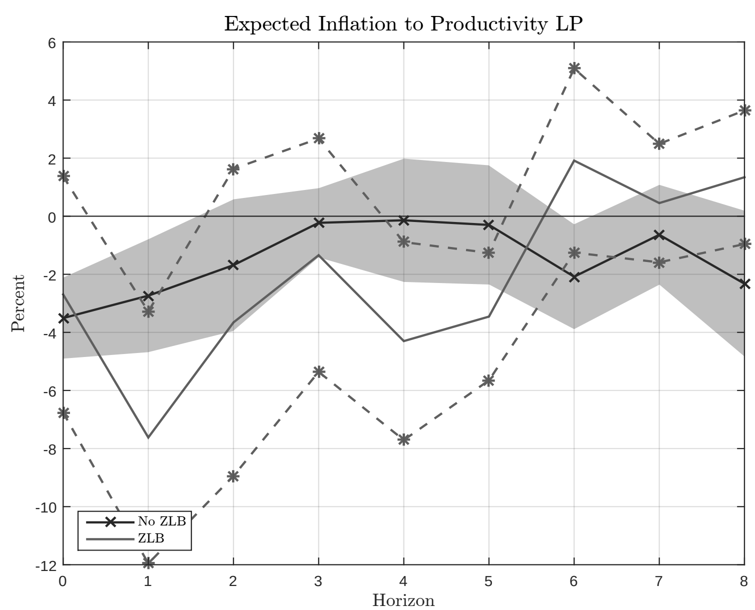


Figure E2: Results from Baseline LP, Expected Inflation

Notes: This figure shows the estimate impulse response of output to a one unit productivity shock at various horizons. The line with 'x' markers covers the case when the ZLB does not bind (i.e. when $Z_t = 0$). The shaded area bands represent the 90 percent confidence interval about the no ZLB case. The solid line shows responses when the ZLB binds or $Z_t = 1$. Theotted lines with '*' markers represent the 90 percent confidence interval about the ZLB case.

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