Technical Appendix for: "Asset Market Participation, Monetary Policy Rules and the Great Inflation"*.

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Abstract

This Appendix illustrates the difficulties encountered when trying to apply the estimation method of Lubik and Schorfheide (2004) to the model in Bilbiie and Straub (2011).

* We are indebted to Roger Farmer, Michel Juillard and Thomas Lubik in particular for useful discussions.

For simplicity, consider the baseline model—without endogenous persistence—outlined in Section 1 of Bilbiie and Straub (2011). The method of Lubik and Schorfheide (2004, henceforth LS) boils down to solving the model consisting of equations—preserving the numbering in the main body of the paper—(2) and (4), having replaced (5), and finding the law of motion of endogenous variables as a function of the fundamental (and, in the case of indeterminacy, sunspot) shocks. The coefficients dictating these laws of motion will be a function of the deep parameters of the model as well as, in the case of indeterminacy, a set of new free parameters. The likelihood function (the joint probability density of the sample of observations, given the parameters) is constructed starting from this model solution (see Section II of LS for details). What is important for our point though is that the model's solution (and hence the likelihood function) depends evidently on the eigenvalues of the dynamic system. But for our model, the eigenvalues (and hence also the likelihood function) are discontinuous in the asset markets participation parameter.

To prove this result formally, we calculate their limits of our system's eigenvalues as the participation parameter λ tends to its threshold value λ^* from below and above, respectively. The eigenvalues are:

$$q_{\pm} = \frac{1}{2}\left[\text{trace} \pm \sqrt{\text{trace}^2 - 4\det}\right],$$

where $\det = \beta^{-1}$;trace$= 1 - (\phi_\pi - 1)\,\kappa\,(\delta\beta)^{-1} + \beta^{-1}$. Note that:

$$\lim_{\lambda \nearrow \lambda^*} \text{trace} = -\text{sgn}\left[\phi_\pi - 1\right] * \infty; \quad \lim_{\lambda \searrow \lambda^*} \text{trace} = \text{sgn}\left(\phi_\pi - 1\right) * \infty$$

Under active policy, we have: $\lim_{\lambda \nearrow \lambda^*}$trace$= -\infty$; $\lim_{\lambda \searrow \lambda^*}$trace$= +\infty$, implying that the limits from below and above of the larger eigenvalue are $\lim_{\lambda \nearrow \lambda^*} \mid q_+ \mid = 0$; $\lim_{\lambda \searrow \lambda^*} \mid q_+ \mid = +\infty$. Under passive policy: $\lim_{\lambda \nearrow \lambda^*}$trace$= +\infty$; $\lim_{\lambda \searrow \lambda^*}$trace$= 0$, which implies $\lim_{\lambda \nearrow \lambda^*} \mid q_+ \mid = +\infty$; $\lim_{\lambda \searrow \lambda^*} \mid q_+ \mid = 0$. Naturally, the opposite holds for the smaller root, i.e. with active policy: $\lim_{\lambda \nearrow \lambda^*} \mid q_- \mid = +\infty$; $\lim_{\lambda \searrow \lambda^*} \mid q_- \mid = 0$, while with passive policy, $\lim_{\lambda \nearrow \lambda^*} \mid q_- \mid = 0$; $\lim_{\lambda \searrow \lambda^*} \mid q_- \mid = +\infty$.

To see an illustration of this result, consider a standard parameterization involving $\beta = 0.99$, $\varphi = 2$, $\theta = 0.75$ and $\varepsilon = 6$; For this parameterization, the threshold value of the degree of LAMP (at which the bifurcation occurs) is $\lambda = 0.375$. The following figures plot the absolute value of the positive and negative eigenvalues, respectively, as a function of the asset market participation parameter. For each plot we consider an active and a passive policy, so that this covers all the parameter regions of interest. It is clear that in all cases, both eigenvalues are discontinuous in the asset market participation parameter, which illustrates why applying standard inference methods that rely on optimizing over the whole parameter space is inappropriate.

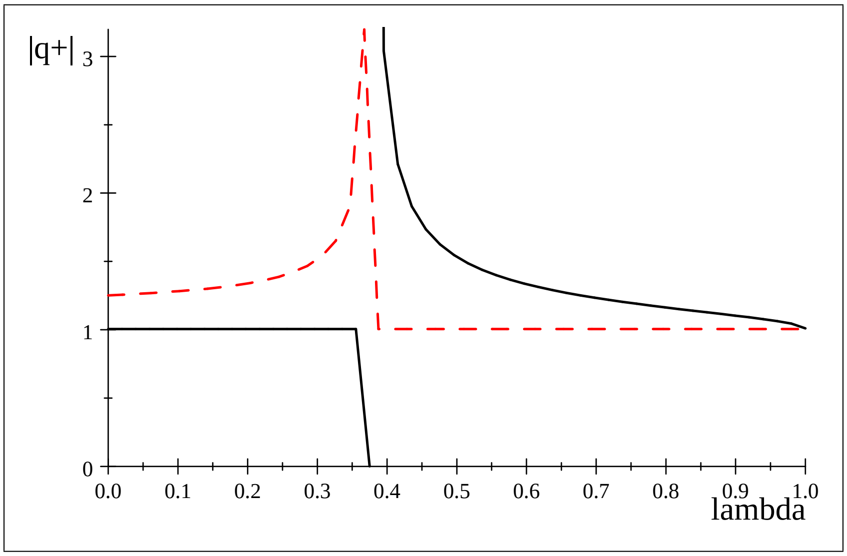

Figure B.1:$| q_+ |$ as a function of λ for $\phi_\pi = 0.8$ (dashed line) and $\phi_\pi = 1.5$ (solid line).

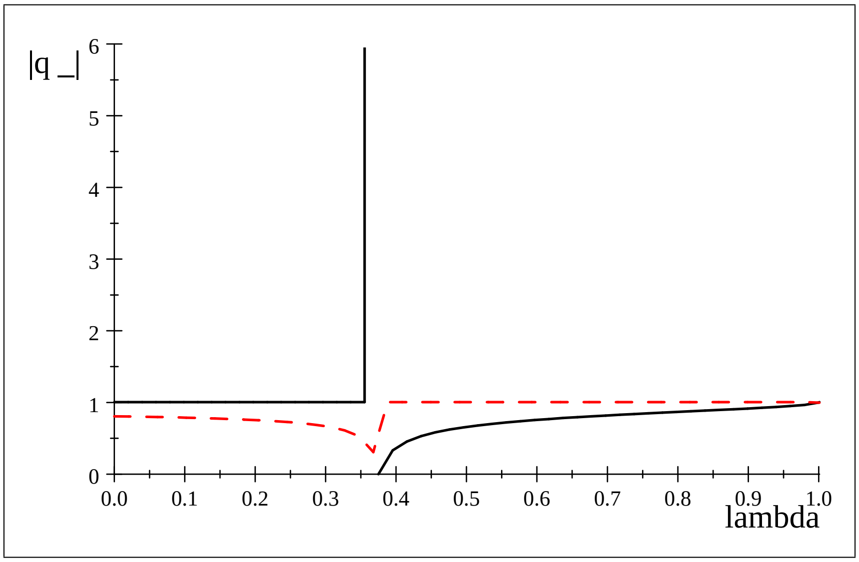

Figure B.2:$| q_- |$ as a function of λ for $\phi_\pi = 0.8$ (dashed line) and $\phi_\pi = 1.5$ (solid line).

It is worth noticing that the type of bifurcation induced by limited asset markets participation is very different from the bifurcation occurring when policy changes from passive to active (which is present also under full participation), precisely because eigenvalues are not discontinuous in the monetary policy response ϕ_π. Indeed, it is clear by mere inspection of the eigenvalues expression that they are continuous in ϕ_π at the bifurcation point $\phi_\pi = 1$:

$$\lim_{\phi_\pi \searrow 1} \mid q_+ \mid = \lim_{\phi_\pi \nearrow 1} \mid q_+ \mid = \beta^{-1}$$
$$\lim_{\phi_\pi \searrow 1} \mid q_- \mid = \lim_{\phi_\pi \nearrow 1} \mid q_- \mid = 1.$$

This holds for any given value of λ (and hence also, trivially, for the full-participation model).

REFERENCES

Bilbiie, F. O. and Straub, R., 2011 "Asset Market Participation, Monetary Policy Rules and the Great Inflation", Forthcoming, *Review of Economics and Statistics.*

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